**C 23880** (Pages : 3) Name.....

Reg. No.....

**SECOND SEMESTER (CBCSS–UG) DEGREE EXAMINATION**
**APRIL 2022**

Mathematics

MAT 2C 02—MATHEMATICS—II

(2020 Admissions)

Time : Two Hours Maximum : 60 Marks

**Section A**

*Answer any number of questions.*
*Each question carries 2 marks.*
*Maximum 20 marks.*

1. Find the inverse of the function $f(x) = \sqrt{3x-2}$.

2. Find the Cartesian form of the polar equation $r = \dfrac{2}{\sin\theta - 2\cos\theta}$.

3. Find the slope of the line tangent to the graph of $r = \cos 3\theta$ at $(r, \theta) = (-1, \pi/3)$.

4. Show that $\lim\limits_{n\to\infty} \dfrac{2n+1}{n} = 2$.

5. Find $\dfrac{dy}{dx}$, where $y = x \sinh x - \cosh x$.

6. Find the norm of the vector $\langle 3, 4, 0, 1, -1\rangle$. Also normalize the vector.

7. Compute $\|\cos x\|$ in $C[0, 2\pi]$.

8. Using Maclaurin's series find the expansion of $\sin x$.

9. Find the determinant of the matrix $C = \begin{bmatrix} 3 & 4 & 8 \\ 2 & -4 & -18 \\ -4 & 7 & 27 \end{bmatrix}$.

**Turn over**

10. Let I be an identity matrix of order $n \times n$ . Show that I is an orthogonal matrix.

11. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ find $A^3$ using Cayley Hamilton theorem.

12. Find the eigen values of the matrix $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$.

## Section B

*Answer any number of questions.*
*Each question carries 5 marks.*
*Maximum 30 marks.*

13. Find the length of the curve $y = (x/2)^{2/3}$ from $y = 0$ to $y = 2$.

14. Find $\left(f^{-1}\right)'(2)$, if $f(x) = x^5 + x$.

15. Find the length of the curve $r = a \sin^2\left(\frac{\theta}{2}\right), 0 \le \theta \le \pi, a > 0$.

16. Evaluate $\int_0^1 e^{-x^2}\, dx$ by means of Trapezoidal rule with $n = 10$.

17. Using Maclaurin's series expand $\tan^{-1} x$. Hence deduce the Gregory series :

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

18. $B_1 = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where $\mathbf{u}_1 = \langle 2, -1, 1\rangle$, $\mathbf{u}_2 = \langle 1, 5, 1\rangle$, $\mathbf{u}_3 = \langle 0, 1, 2\rangle$, is a basis for $\mathbb{R}^2$. Transform it into an orthonormal basis $B_2 = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$.

19. Find the inverse of $\begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$ if it exists.

## Section C

*Answer any* **one** *question.*
*Each question carries* 10 *marks.*
*Maximum* 10 *marks.*

20. (a) Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

(b) For which values of $r$ is $\int_0^1 x^r \, dx$ convergent ? Justify your answer.

21. (a) Solve the system of equations :

$$\begin{aligned} x_1 - 2x_2 + x_5 - x_6 + x_7 &= 0 \\ x_3 - x_4 + x_5 - 2x_6 + 3x_7 &= 0 \\ x_1 - x_5 + 2x_6 &= 0 \\ 2x_1 - 3x_4 + x_5 &= 0. \end{aligned}$$

(b) Diagonalize the matrix $A = \begin{bmatrix} 10 & 3 \\ 4 & 6 \end{bmatrix}$.