18/08/2023

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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2023

(CBCSS)

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

*Answer **all** questions.*
Each question carries 1 weightage.

1. Define base of a topological space. Illustrate with an example.
2. Prove that a discrete space is second countable iff the underlying set is countable.
3. Let $(\mathbf{X}, \mathfrak{T})$ be a topological space and let $A \subset \mathbf{X}$. Then prove that $\overline{\overline{A}} = \overline{A}$.
4. Define a local base at a point x in a space **X.** Give an example. When will you say that a space is first countable ?
5. When will you say that a space is connected ? Prove that if **X** is connected then **X** cannot be written as the disjoint union of two nonempty closed sets.
6. In a Hausdorff space, prove that limits of sequences are unique.
7. State Urysohn's lemma.
8. Prove that every compact Hausdorff space is a $\mathbf{T}_3$ space.

(8 × 1 = 8 weightage)

Turn over

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Part B

Answer any **two** *questions from each unit.*
Each question carries 2 weightage.

Unit 1

9 Prove that the real line with the semi-open interval topology is not second countable.

10 Let $(\mathbf{X}, \mathfrak{T})$ be a topological space and let $\mathfrak{B} \subset \mathfrak{T}$. Then prove that $\mathfrak{B}$ is a base for $\mathfrak{T}$ iff for any $x \in X$ and any open set G containing x, there exist $b \in \mathfrak{B}$ such that $x \in G$ and $B \subset G$.

11 Let $(\mathbf{X}, \mathfrak{T})$ and $(\mathbf{Y}, \mathfrak{U})$ be topological spaces and let $f : X \to Y$ be any function. If f is continuous at $x_0 \in X$, then prove that for every subset A of X, $x_0 \in \overline{A}$ implies $f(x_0) \in \overline{f(A)}$.

Unit 2

12 Prove that the co-countable topology on a set makes it into a Lindeloff space.

13 Prove that the product topology is the weak topology determined by the projection functions.

14 Prove that every closed, surjective map is a quotient map.

Unit 3

15 Suppose a topological space has the property that for every closed subset A of X, every continuous real valued function on A has a continuous extension to it. Then prove that X is normal.

16 Prove that all metric spaces are T_4.

17 Suppose $\boldsymbol{y}$ is an accumulation point of a subspace **A** of a $\mathbf{T_1}$ space **X**. Then every neighbourhood of $\boldsymbol{y}$ contains infinitely many points of **A**.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *from the following four questions.*
Each question carries 5 weightage.

18. (a) For a subset A of a topological space X, prove that $\overline{A} = A \cup A'$.

(b) Define sub-base for a topology. Let $(\mathbf{X}, \mathfrak{T})$ be a topological space and **S** a family of subsets of **X.** Then **S** is a sub-base for $\mathfrak{T}$ if and only if **S** generates $\mathfrak{T}$.

19. (a) The topological product of a finite number of connected spaces is connected. Prove the statement.

    (b) Prove that every closed and bounded interval is compact.

20. (a) For a topological space X, prove that the following statements are equivalent :

    (i) X is locally connected.

    (ii) Components of open subsets of X are open in X.

    (iii) X has a base consisting of connected subsets.

    (iv) For every $x \in X$ and every neighbourhood N of x, there exist a connected open neighbourhood M of x such that $M \subset N$.

    (b) Prove that every quotient space of a locally connected space is locally connected.

21. (a) Prove that the axioms T_0, T_1, T_2, T_3 and T_4 form a hierarchy of progressively stronger conditions.

    (b) Give an example of :

    (i) a T_0 space which is not T_1.

    (ii) a T_1 space which is not T_2.

    (iii) a T_2 space which is not T_3.

(2 × 5 = 10 weightage)