D 51447 (Pages : 3) Name..............................

Reg. No..............................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCSS—PG)

Mathematics

MT 1C 05—DISCRETE MATHEMATICS

(2016 Syllabus Year)

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries weightage 1.

1. In any graph on n vertices, show that the number of vertices of odd degree is even.

2. If G is a simple graph on n vertices and $\delta \geq \frac{n-1}{2}$, prove that G is connected.

3. Define join of two graphs. Write $K_{m,n}$ as the join of two graphs.

4. Define a k-vertex cut of a graph. Give a non-trivial example.

5. Prove that an edge $e = xy$ is a cut edge of a connected graph G if there exists vertices u and v such that e belongs to every $u - v$ path.

6. Define a maximal planar graph. Give an example.

7. Define : (i) Irreflexive order ; and (ii) Asymetric order. Give examples for each of these order.

8. Let $(X, \leq)$ be a partially ordered set and $x, y \in X$. Define the condition for y to cover x. Give an example.

9. If $(X_1, \leq_1)$ and $(X_2, \leq_2)$ are partially ordered sets, define a partial order $\leq$ on (X_1, X_2). Verify whether $\leq$ is total if $\leq_1$ and $\leq_2$ are total.

10. Define a Boolean function of k variables. Give an example of a Boolean function of 2 variables.

11. Let $\Sigma = \{a, b, c\}$. If $L = \{a, b\}$, find L^+, L^R and L^*.

Turn over

12. Design a *dfa* which accepts string 00 only.

13. If $\Sigma = \{0, 1\}$, design an *nfa* to accept set of strings ending with two consecutive zeros.

14. Let $\Sigma = \{a, b\}$. Find an *dfa* for the language $L = \{\omega : |\omega| \bmod 3 = 0\}$.

(14 × 1 = 14 weightage)

Part B (Paragraph Type)

Answer any **seven** *questions from the following ten questions.*
Each question carries weightage 2.

15. Show that $Aut(K_n) \approx S_n$, for any $n \in \mathbb{N}$.

16. If $\{x, y\}$ is a 2-edge cut set of a graph G, show that every cycle of G that contains x must also contain y.

17. If a graph G with at least 3 vertices is 2- connected prove that any *two* vertices of G are connected by at least two internally disjoint paths.

18. Prove that a graph is planar if and only if it is embeddable on sphere.

19. Prove that K_5 is non-planar.

20. Find the conjunctive normal form of the function $a'b(a' + b + ab)$.

21. Let $(X, +, \cdot, ')$ be a Boolean algebra. Prove that $x + x \cdot y = x$ for all $x, y \in X$.

22. Prepare the table for values of the function $f(x_1, x_2, x_3) = x_1x_2 + x_1'x_3$.

23. Find a *dfa* that accepts all strings on $\{0, 1\}$ except those containing the substring 001.

24. Find a grammar that generates $L = \{a^n b^{n+1} : n > 0\}$ on $\Sigma = \{a, b\}$.

(7 × 2 = 14 weightage)

Part C (Essay Type)

Answer any **two** *questions from the following four questions.*
Each question carries weightage 4.

25. Prove that the following statements are equivalent :—

(a) G is connected and unicyclic.

(b) G is connected and $n = m$.

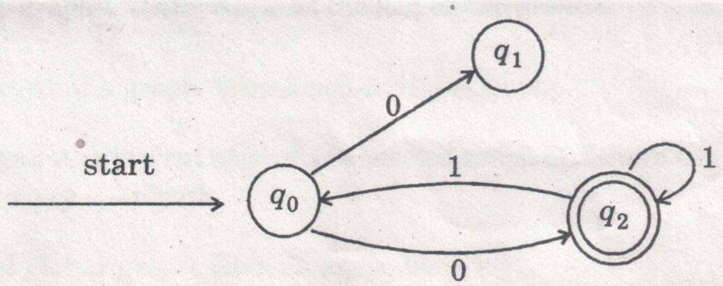
(c) For some edge e of G, G $- e$ is a tree.

(d) G is connected and the set of edges of G that are not cut edges forms a cycle.

26. Show that $k(G) \leq \lambda(G) \leq \delta(G)$ if G is a simple connected graph.

27. Prove that $(X, \leq)$ is a lattice where $(X, +, \cdot, ')$ is the Boolean algebra and $\leq$ is defined in X by $x \leq y$ if and only if $x \cdot y' = 0$. Find the maximum and minimum elements of this lattice.

28. Convert the *nfa* given by the transition graph into an equivalent *dfa* :



(2 × 4 = 8 weightage)