C 32330 (Pages : 2) Name.....................................

Reg. No.....................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA—I

(2010 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*

Each question carries weightage 1.

1. Verify whether $\phi(x, y) = (x + 1, y + 2)$ is an isometry of the plane.
2. Find the order of (2, 3) in the group $\mathbb{Z}_4 \times \mathbb{Z}_6$.
3. Give two non isomorphic abelian groups of order 8.
4. Find the weight of the binary code u = 00110001.
5. Give a composition series of the group $\mathbb{Z}_{12}$.
6. Find the commutator subgroup of S_3.
7. Let H be a subgroup of a group G and X be the set of all left cosets of H in G which is a G-set with $g.(x\text{H}) = (gx)\,\text{H}$. Find the orbit of H.
8. Find the number of 5-Sylow subgroups of a group G where $|\text{G}| = 50$.
9. Show that all groups of order 15 contains a normal subgroup of order 5.
10. Let F (x, y) be the free group on $\{x, y\}$. Give a homomorphism from F (x, y) to $\mathbb{Z}_2$.
11. Give a presentation of $\mathbb{Z}_2 \times \mathbb{Z}_3$ using one generator.
12. Verify whether $x^3 + x^2 + 2x + 2$ is irreducible in $\mathbb{Z}_3[x]$.
13. Find the inverse of $(1 + 2i + j)$ in the ring of quaternions.
14. Verify whether N = {0, 1, 2} is an ideal of the ring $\mathbb{Z}_6$.

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Verify whether $\mathbb{Z}_2 \times \mathbb{Z}_2$ is isomorphic to the Klein four group.
16. Find the kernel of the homomorphism $\eta : G \to G/N$ defined by $x \mapsto Nx$.
17. Let N be a maximal normal subgroup of a group G. Show that G/N is a simple group.
18. Let H, K be groups. Show that $\bar{H} = \{(h, e) \in H \times K : h \in H\}$ is a normal subgroup of $H \times K$ where e is the identity of K.
19. Show that every group of order 35 has a normal subgroup of order 7.
20. Show that if H and N are subgroups of a group G and if N is normal then HN is a subgroup of G.
21. Verify whether $\mathbb{Z} \times \mathbb{Z}$ is a free group.
22. Give a presentation of the group $\mathbb{Z}_2 \times \mathbb{Z}_4$ using two generators.
23. Let $\phi_\alpha : \mathbb{Q}[x] \to \mathbb{Q}$ be the evaluation homomorphism where $f(x) \mapsto f(\alpha)$. Find ker ϕ_α.
24. Verify whether $x^5 - 12x^3 + 12$ is irreducible in $\mathbb{Z}[x]$.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. (a) Define isomorphic subnormal series of a group.

    (b) Show that any two subnormal series of a group G have isomorphic refinements.

26. (a) Define commutator subgroup of a group.

    (b) Show that if K is the commutator subgroup of G then :

    (i) K is a normal subgroup of G.

    (ii) G/K is abelian.

27. (a) Describe the free group on a set A.

    (b) Prove that if F is the free group on Aand G is any group then every map $f : A \to G$ extends to a homomorphism from F to G.

28. (a) Describe the group ring R (G) of a group G over a ring R.

    (b) Show that R (G) is a ring.

    (c) List all elements of the groupring R (G) for $R = \mathbb{Z}_3$ and G $\mathbb{Z}_2$.

(2 × 4 = 8 weightage)