

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

*Answer all the twelve questions.**Each question carries 1 mark.*

1. What is the period of $f(z) = e^{2iz}$?
2. Give an example of an entire function.
3. What is the complex, form of Cauchy-Riemann equations ?
4. Define entire function.
5. What are the singularities of $f(z) = |z|^2$.
6. State Cauchy's integral formula.
7. State Morera's theorem.
8. State Gauss mean value theorem.
9. Define Residue of a complex function.
10. Give an example of an essential singularity.
11. Define removable singularity of a complex function.
12. What, is the residue at a removable singularity ?

(12 × 1 = 12 marks)

Section B

*Answer any ten out of fourteen questions.**Each question carries 4 marks.*

13. Define Analytic functions. Give an example.
14. Show that $(z) = \sin x \cos h y + i \cos x \sin h y$ is an entire function.

Turn over

15. If $f = u + iv$ is analytic, then show that u and v are harmonic.
16. Prove or disprove: $\text{Log}(a^b) = b\text{Log}(a)$, where Log is the principal branch of logarithm and $a, b \in \mathbb{C}$.
17. State Cauchy's integral formula and its extension.
18. Find all the values of $\sin^{-1}(-i)$.
19. Is Cauchy-Goursat theorem valid for arbitrary connected domains? Prove your claim.
20. Using Liouville's theorem prove the fundamental theorem of algebra.
21. Evaluate $\int_C \frac{e^{-z} dz}{z - i\pi/2}$, where C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$.
22. Suppose $z_n = x_n + iy_n$, $n = 1, 2, 3, \dots$, and $S = X + iY$. If $\sum_{n=1}^{\infty} z_n = S$, then show that $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$.
23. State Laurent theorem.
24. Give an example of a non isolated singularity.
25. Using Cauchy's integral theorem, evaluate $\int_C \frac{z+1}{z^2-2z} dz$, where C is the circle $|z| = 3$ in the positive sense.
26. Define pole and its order of a complex function.

(10 × 4 = 40 marks)

Section C

Answer any **six out of nine** questions.

Each question carries 7 marks.

27. Derive Cauchy-Riemann equations.
28. If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D , then show that u and v are harmonic in D .

29. Find the harmonic conjugate of $u(x, y) = \sin h x \sin y$.
30. Using contour integration, evaluate $\int_C z^{1/2} dz$, where C is the path given by $z = 3e^{i\theta}$, $0 \leq \theta \leq \pi$.
31. State and prove the principle of domination of paths.
32. Find the Laurent series that represents the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ in the domain $0 < |z| < \infty$.
33. If a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges when $z = z_1$ ($z_1 \neq z_0$) then show that it is absolutely convergent at each point z in the open disk $|z - z_0| < R_1$, where $R_1 = |z_1 - z_0|$.
34. State- and prove Cauchy's residue theorem.
35. Using residue evaluate $\int_0^{\infty} \frac{dx}{x^4 + 1}$.

(6 × 7 = 42 marks)

Section D

Answer any two out of three questions.

Each question carries 13 marks.

36. State and prove reflection principle.
37. (a) State and prove Liouville's Theorem.
 (b) Using Liouville's theorem, prove fundamental theorem of algebra.
38. (a) Show that the power series $S(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ is analytic each point, z interior to the circle of convergence of that series.
 (b) Find the residue of $\frac{1}{z + z^2}$ at $z = 0$.

(2 × 13 = 26 marks)