D 31818 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(2019 Admission Onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions from this section.
Each question carries 2 marks.
Maximum marks : 25.

1. Find the derivative of $\log \sqrt{x^2+1}$.

2. Find the derivative of $\tan^{-1} \sqrt{2x+3}$.

3. Evaluate $\lim\limits_{x \to \infty} \dfrac{\log x}{x}$.

4. Let $f(x) = e^x + x$. (a) find the derivative of f ; (b) find an equation to the tangent line to the graph of $f(x)$ at $x = 0$.

5. Evaluate $\int\limits_{-1}^{\infty} e^{-x} dx$.

6. Determine whether $\left\{\dfrac{n}{n+1}\right\}$ converges or diverges.

7. Determine whether the series $\sum\limits_{n=1}^{\infty} 3\left(\dfrac{-1}{2}\right)^{n-1}$ converges or diverges. If it converges, find the sum.

8. What is an alternating series ? Give an example.

9. Define a power series. Give an example.

Turn over

10. Find the Maclaurin's series of $f(x) = e^x$ and determine its radius of convergence.

11. Find $\frac{d^2y}{dx^2}$ if $x = t^2 - u$ and $y = t^3 - 3t$.

12. Find the parametric equation for a line L passing through the points $P(-3,3,-2)$ and $G(2,-1,4)$.

13. Find an equation in rectangular co-ordinates for the surface with the given cylindrical equation $r^2 \cos 2\theta - z^2 = 4$.

14. Find the point of tangency and unit tangent vector at the point on the curve :

$r(t) = (t^2 + 1)i + e^{-t} j - \sin 2tk$ at $t = 0$.

15. Find the length of the arc of the helix given by $r(t) = 2\cos ti + 2\sin tj + tk, 0 \le t \le 2\pi$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum marks : 35.

16. Find the derivative of $y = \frac{(2x-1)^3}{\sqrt{3x+1}}$.

17. Find $\int \cosh^2(3x)\sinh(3x)\,dx$.

18. Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$.

19. (a) Use integral test to determine whether the series $\sum_{n=1}^{\infty} \frac{\log n}{n}$ converges or diverges.

(b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2+2}$ converges or diverges.

20. (a) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

(b) Find a power series representation of $\log(1-x)$ on $(-1, 1)$.

21. Sketch the curve described by the parametric equations $x = t^2 - 4, y = 2t, -1 \le t \le 2$.

22. Find an equation of the plane containing the points $P(3,-1,1)$, $Q(1,4,2)$ and $R(0,1,4)$.

23. Find the curvature of the twisted cubic described by the vector function $r(t) = ti + \frac{1}{2}t^2 j + \frac{1}{3}t^3 k$.

Section C

Answer any number of questions from this section.
Each question carries 10 marks.
Maximum marks : 20.

24. (a) Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$.

(b) A power line is suspended between two towers. The shape of the cable is a catenary with equation $y = 80 \cosh \frac{x}{80}, -100 \le x \le 100$, where x is measured in feet. Find the length of the cable.

25. (a) Show that $\int_0^{\infty} e^{-x^2} dx$ is convergent.

(b) Find $\lim_{n \to \infty} \frac{n!}{n^n}$.

26. (a) Find the Taylor series for $f(x) = \sin x$ at $x = \pi/6$.

(b) Find the area of the region enclosed by the cardioid $r = 1 + \cos\theta$.

27. (a) Identify and sketch the surface $12x^2 - 3y^2 + 4z^2 + 12 = 0$.

(b) A particle moves along a curve described by the vector function $r(t) = ti + t^2 j + t^3 k$. Find the tangential scalar and normal scalar components of acceleration of the particle at time t.