C 24738 (Pages : 4) Name..................................

Reg. No..................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2017

(CUCBCSS—UG)

Core Course—Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Find dy if $y = \frac{2x}{1+x^2}$.

2. A function with a continuous first derivative is said to be ————.

3. Suppose that $\int_1^3 f(x)\,dx = 6$. Find $\int_1^3 f(u)\,du$.

4. If f is smooth in $[a, b]$ then the length of the curve $y = f(x)$ from a to b is L = ————.

5. Find the intervals in which the function f is increasing given $f'(x) = x(x-1)$.

6. The radius r of a circle increases from $r_0 = 10m$ to $10.1m$. Estimate the increase in the circle's area A by calculating dA.

7. Evaluate $\int_0^1 \left(x^2 + \sqrt{x}\right) dx$.

8. Write the sum without sigma notation and then evaluate the sum $\sum_{k=1}^{4} \cos k\pi$.

9. State Rolle's Theorem.

10. What are the critical points of f given $f'(x) = x^{-1/3}(x+2)$.

Turn over

11. Evaluate $\lim_{x \to \infty} \frac{\sin 2x}{x}$.

12. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find the absolute maximum and minimum values of $f(x) = -\frac{1}{x}, -2 \leq x \leq -1$.

14. Evaluate $\int_0^{\pi/4} \tan x \sec^2 x \, dx$.

15. Find the volume of the solid generated by revolving the region bounded by the line $y = 0$ and the curve $y = x - x^2$.

16. Suppose that f is continuous and that $\int_0^3 f(x)\,dx = 3$ and $\int_0^4 f(x)\,dx = 7$. Find $\int_4^3 f(x)\,dx$.

17. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point (0, 2).

18. Find the average value of $f(x) = x^2 - 1$ on $(0, \sqrt{3})$.

19. Evaluate $\sum_{k=1}^{7} (-2k)$.

20. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.

21. Show that if *f* is continuous on $[a, b]\ a \neq b$ and if $\int_a^b f(x)dx = 0$ then $f(x) = 0$ at least once in $[a, b]$.

22. Evaluate $\frac{d}{dt}\int_0^{t^4} \sqrt{u}\, du.$

23. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\frac{\pi}{4}$.

24. Express the solution of the following initial value problem as an integral :

Differential equation : $\frac{dy}{dx} = \tan x$

Initial condition : $y(1) = 5.$

(9 × 2 = 18 marks)

Part C

Answer any six questions.
Each question carries 5 marks.

25. Find the lateral surface area generated by revolving $xy = 1,\ 1 \leq y \leq 2$ about the *y*-axis.

26. About how accurately should we measure the radius *r* of a sphere to calculate the surface area $S = 4\pi r^2$ within 1% of its true value.

27. Evaluate the length of the curve $x = \sqrt{1 - y^2},\ -\frac{1}{2} \leq y \leq \frac{1}{2}$.

28. Find the volume of the solid generated by revolving the region between the *y*-axis and the curve $x = \frac{2}{y}, 1 \leq y \leq 4$ about the *y*-axis.

29. Find the asymptotes of the curve $y = \frac{x+3}{x+2}$.

30. Find the intervals on which the function $h(x) = -x^3 + 2x^2$ is increasing and decreasing.

31. Find the length of the curve $x = \sin y, 0 \le y \le \pi$.

32. Find the area of the region enclosed by the curve $y = x^2 - 2$ and the line $y = 2$.

33. Find the value of local maxima and minima of $f(x) = x^2 - 4, -2 \le x \le 2$ and $2ay$ where they are assumed.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

35. State and prove the Fundamental Theorem of calculus.

36. Find the centre of mass of a thin plate of constant density δ covering the region bounded by the parabola $y = 4 - x^2$ and below by the x-axis.

(2 × 10 = 20 marks)