

FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all twelve questions.

1. Find $\lim_{x \rightarrow 2} \frac{-x}{(x-3)^2}$.
2. Differentiate $\cos(x^2 + 2)$ with respect to x .
3. Suppose $\lim_{x \rightarrow c} f(x) = 5$ and $\lim_{x \rightarrow c} g(x) = -2$. Find $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$.
4. Find second derivative of $y = \sin x + \cos x$.
5. Define a decreasing function.
6. Evaluate $\sum_{k=1}^{10} k^2$.
7. Find $\lim_{x \rightarrow \infty} \frac{\pi \sqrt{3}}{x^2}$.
8. Absolute maximum of the function $y = x^2$ on $[0, 2]$ is _____.
9. Find dy if $y = x^5 + 37x$.
10. Define critical point of a function.

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11. Define norm of a partition.
12. Suppose that $\int_1^4 f(x) dx = -2$. Evaluate $\int_4^1 f(x) dx$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** questions.

13. Evaluate $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$.
14. Find absolute extremes of $h(x) = x^{2/3}$ on $[-2, 3]$.
15. Find $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t dt$.
16. Given $1 - x^2/4 \leq u(x) \leq 1 + x^2/2$, for all $x \neq 0$. Find $\lim_{x \rightarrow 0} u(x)$.
17. If $\lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 1$, find $\lim_{x \rightarrow -2} \frac{f(x)}{x^3}$.
18. Show that $\lim_{x \rightarrow 1} 5x - 3 = 2$.
19. Evaluate $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.
20. Find the derivative of $y = \frac{x^2 + 1}{x^2 - 1}$.
21. Find the linearization of $x^3 - x$ at $x = 1$.

22. Express the limit $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n (c_k^2 - 3c_k) \Delta x_k$ as definite integral where P is the partition of $[-7, 5]$.
23. Give an example of a function with no Riemann integral. Explain.
24. Show that the value of $\int_0^1 \sqrt{1 + \cos x} \, dx$ cannot possibly be 2.

(9 × 2 = 18 marks)

Part C (Short Essay Type)*Answer any six questions.*

25. Find $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$.
26. Find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.
27. Find the interval on which $g(x) = -x^3 + 12x + 5$, $-3 \leq x \leq 3$ is increasing and decreasing. Where does the function assume extreme values and what are these values?
28. Show that the functions with zero derivatives are constant.
29. Show that $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$ has a continuous extension to $x = 2$, and find that extension.
30. For what values of a is $f(x) = \begin{cases} x, & x < -2; \\ ax^2, & x \geq -2 \end{cases}$ continuous at every x ?
31. Find the slope of the parabola $y = x^2$ at the point P (2, 4). Write an equation for the tangent to the parabola at this point.
32. Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so where?
33. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$, about the x -axis.

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)*Answer any two questions.*

34. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5}y^2$, $x = 0$, $y = -1$, $y = 1$ about x -axis.

35. Let $f(x) = \begin{cases} 3-x, & x < 2; \\ \frac{x}{2} + 1, & x > 2. \end{cases}$

a) Find $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$.

b) Does $\lim_{x \rightarrow 2} f(x)$ exist? If so, what is it? If not, why not?

c) Find $\lim_{x \rightarrow 4^+} f(x)$ and $\lim_{x \rightarrow 4^-} f(x)$.

d) Does $\lim_{x \rightarrow 4} f(x)$ exist? If so, what is it? If not, why not?

36. Find the center of mass of a thin plate of constant density δ covering the region bounded above by parabola $y = 4 - x^2$ and below by x -axis.

(2 × 10 = 20 marks)