D 31179 (Pages : 2) Name..................................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2022

(November 2021 Session for SDE/Private Students)

(CBCSS)

Mathematics

MTH 3C 13—FUNCTIONAL ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Find the codimension of the subspace c_0 inside the space c of convergent sequences.
2. Justify the statement: All normed spaces are metric spaces.
3. If X_0 is a closed space of a linear space X then prove that the quotient space X/X_0 is a normal space.
4. State and prove parallelogram law.
5. Let H be an inner product space. Show that H is also a normed space under the Hilbertian norm.
6. State the Banach open theorem.
7. Show that for a normed space X, X* is a total set.
8. Show that for the shift operator A in l_2 A^n does not converge strongly to zero.

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions choosing two from each unit.*
Each question carries weightage 2.

UNIT 1

9. Prove that the intersection of convex sets is convex.
10. Show that l_p, $1 \leq p \leq \infty$ is a Banach space.
11. State and prove the integral form of Minkowski inequality.

Turn over

UNIT 2

12. State and prove a condition for a Hilbert space to be separable.

13. Show that for every closed subspace L of Hilbert space $H\ L \oplus L^{\perp} = H$ and $(L^{\perp})^{\perp} = H$.

14. State and Prove Bessel's Inequality.

UNIT 3

15. Show that a Banach space X can be identified with a subspace of X**.

16. Let X, Y be any *two* Banach spaces. Prove that for a linear operator $A : X \to Y$ implies $A^*: X^* \to {}^*Y^*$ is compact.

17. State and prove any *two* properties of invertible operators.

(6 × 2 = 12 weightage)

Part C

Answer **two** *questions.*
Each question carries weightage 5.

18. State and prove Cauchy Schwartz inequality.

19. (a) Explain Gram-Schmidt orthogonalisation procedure.

    (b) State and prove Riesz representation theorem.

20. Show that L (X, Y) is a Banach space.

21. Show that the set $K(X \to Y)$ of compact operators from X to Y satisfy.

    (i) $K(X \to Y)$ is a linear sub-space of $L(X \to Y)$

    (ii) If $A \in K(X \to Y)$, $B \in L(Z \to X)$, $C \in L(Y \to Z)$ then $AB \in K(Z \to Y)$, $CA \in K(X \to Z)$

    (iii) $K(X \to Y)$ is a closed subspace of $L(X \to Y)$.

(2 × 5 = 10 weightage)