**D 73128** (Pages : 4) Name.......................................

Reg. No.......................................

# FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

## MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

### Part A (Objective Type)

*Answer all* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. At what points are the function $f(x) = \frac{1}{x-2} - 3x$ continuous ?
2. Define an equivalence relation.
3. Find $n(A \times B)$ for the sets $A = \{1, 2\}$ and $B = \{a, b, c\}$.
4. Let $f : R \to R$ be defined by $f(x) = x^2 + 2x$. Find $f \circ f(2)$.
5. Find the domain of the function $f(x) = \frac{1}{x-2}$.
6. The graph of $y = x^2$ is shifted 2 units left and 2 units up, write the equation for the new graph.
7. Let $S = \{1, 2, 3\}$. Find the number of elements of power set of S.
8. Find the number of constant functions from A into B.
9. Translate these statement "$\forall x (C(x) \to F(x))$" into English, where $C(x)$ is "$x$ is a comedian" and $F(x)$ is "$x$ is funny" and the domain consists of all people.
10. Consider the sets $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6, 7\}$. Find $A \oplus B$.
11. If $A \subseteq B$ and $A \cap B = A$, then $A \cup B =$ .........
12. Suppose $f : A \to B$ is a constant function. When will $f$ be one-to one ?

(12 × 1 = 12 marks)

Turn over

## Part B (Short Answer Type)

*Answer any **nine** questions.*

*Each question carries 2 marks.*

13. Evaluate $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$.

14. Check whether the function $f(x) = x^2 - 2$ is bijective.

15. Find $x$ and $y$ if $(y-2, 2x+1) = (x-1, y+2)$.

16. Let $P(x)$ denote the statement "$x > 3$." What are the truth values of $P(4)$ and $P(2)$?

17. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Find the number of functions from :

    (a) A into B ; (b) B into A.

18. Let $X = \{1, 2, 3, 4\}$. Determine whether the relation $\{(2,3), (1,4), (2,1), (3,2), (4,4)\}$ from X to X is a function.

19. If $\lim_{x \to -2} \frac{f(x)}{x^2} = 1$, find $\lim_{x \to -2} f(x)$ and $\lim_{x \to -2} \frac{f(x)}{x}$.

20. Define $f(3)$ in a way that extends $f(x) = \frac{x^2 - 9}{x - 3}$ to be continuous at $x = 3$.

21. For what vales of $a$ is $f(x) = \begin{cases} x^2 - 1, & x < 3; \\ ax, & x \geq 3; \end{cases}$ continuous at every $x$ ?

22. Consider the functions $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$. Find a formula defining the composition function : (a) $f \circ g$; (b) $g \circ f$.

23. Prove $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

24. Give examples of relations R on $A = \{1, 2, 3\}$ having the stated property :

(a) R is both symmetric and antisymmetric ;

(b) R is neither symmetric nor antisymmetric.

($9 \times 2 = 18$ marks)

## Part C (Short Essay Type)

*Answer any* **six** *questions.*
*Each question carries 5 marks.*

25. Find $\lim_{h \to 0} \dfrac{\sqrt{2+h} - \sqrt{2}}{h}$.

26. Let $V = \{1, 2, 3, 4\}$ and let $f = \{(1,3), (2,1), (3,4), (4,3)\}$ and $g = \{(1,2), (2,3), (3,1), (4,1)\}$. Find :

(a) $f \circ g$ ; (b) $g \circ f$ ; (c) $f \circ f$.

27. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

28. Show that $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

29. Let R be the relation on positive integers defined by the equation $x + 3y = 12$ :

(a) Write R as a set of ordered pairs.

(b) Find : (i) Domain of R ; (ii) Range of R ; (iii) $R^{-1}$.

(c) Find the composition relation $R \circ R$.

30. Suppose $\zeta$ is a collection of relations S on a set A and let T be the intersection of the relations S, that is, $T = \cap(S : S \in \zeta)$. Prove that if every S is transitive, then T is transitive.

31. Use quantifiers-and predicates to express the fact that $\lim_{x \to a} f(x)$ does not exist.

32. Prove that a function $f : A \rightarrow B$ is invertible if and only if $f$ is objective.

33. Let $A=\{1,2,3\}, B=\{a,b,c\}, C=\{x,y,z\}$. Consider the following relation R from A to B relation S from B to C : $R=\{(1,b),(2,a),(2,c)\}$ and $S=\{(a,y),(b,x),(c,y),(c,z)\}$.

(a) Find the composition relation $R \circ S$.

(b) Find the matrices $M_R, M_S$ and $M_{R \circ S}$ of the respective relations $R, S$ and $R \circ S$; and compare $M_{R \circ S}$ to the product $M_R M_S$.

(6 × 5 = 30 marks)

## Part D (Essay Type)

*Answer any* **two** *questions.*

34. Let $f(x)=\begin{cases} \sqrt{1-x^2}, & 0 \le x < 1 ; \\ 1, & 1 \le x < 2 ; \\ 2, & x = 2 \end{cases}$.

(a) What are the domain and range of $f$?

(b) At what points c, if any, does $\lim_{x \to c} f(x)$ exist?

(c) At what points does only the left-hand limit exists?

(d) At what points does only the right-hand limit exists?

35. Let $S=\{1,2,3,....,19,20\}$. Let $\equiv$ be the equivalence relation on S defined by congruence modulo 7.

(a) Find the quotient set $S/\equiv$.

(b) Find a system of equivalence class representatives consisting of even integers.

36. Prove that "If $n$ is a positive integer, then $n$ is odd if and only if $n^2$ is odd.

(2 × 10 = 20 marks)