C 60049 (Pages : 4) Name..

Reg. No..................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2019

(CUCBCSS)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Solve for $z : 5z = 2i\bar{z}$.
2. State Cauchy-Goursat theorem with full assumptions involved.
3. Verify whether $f(z) = \bar{z}$ is analytic or not at $z = 0$.
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z-2)^2}{z^3\left(z^2+1\right)}$.
5. Is $u(x, y) = x^2 + y^2 - xy$ a harmonic function ? Justify your claim.
6. Define a simply connected domain.
7. Fill in the blanks : The real part of $\cosh(2z)$ is ————.
8. Fill in the blanks : The locus of the points z satisfying $|z - 2i| = 2|i - 1|$ is a/an ————.
9. If an infinite series of complex numbers converges, then show that its n^{th} term converges to zero.
10. If R is the radius of convergence of $\sum a_n z^n$, find the radius of convergence of $\sum n^2 a_n z^n$.
11. What do you mean by a contour ?
12. Find i^i.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Find the real and imaginary parts of the function $f(z) = \log(z)$.

14. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.

15. Show that $\tan^{-1}(z) = \frac{i}{2}\log\frac{i+z}{i-z}$.

16. Show that the zeros of an analytic function are isolated.

17. Evaluate the line integral of $f(z) = z^2$ over the line joining i to $2i - 1$.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty}\frac{n!\,z^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^2$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Locate the poles and zeros, if any, of $f(z) = \cos(1/z)$ in the complex plane.

21. Find all the solutions of $e^z = 3$.

22. Find the residue of $f(z) = \sin(z)/z^2$ at $z = 0$ and evaluate the integral of $f(z)$ around the ellipse containing zero inside it.

23. Using the definition of continuity show that the composite of two continuous functions is continuous.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Which one is bigger : $\| z_1 | - | z_2 \|$ or $| z_1 - z_2 |$. Prove your claim.

26. Determine all the poles of the $f(z) = \sec^2 z$ lying in the disc $| z - \pi/2 | \leq 2$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks.*

27. Determine the nature of the singularities of the function $f(z) = \sin(1/z)$. Does this function have zeros ? Find them if any.

28. Evaluate $\oint_C \frac{z}{(z-a)(z-b)}$ discussing the cases of containment of the points $a \neq 0$ and $b \neq 0$ inside and outside the simple closed curve C.

29. Find the Laurentz series expansion of $f(z) = \frac{z}{(z-1)^2(z-2)}$ discussing the various regions of validity for the expansion.

30. Prove the converse of Cauchy-Goursat's integral theorem by fully stating the assumptions involved.

31. Find the harmonic conjugate of $u(x, y) = e^x(x\cos y - y\sin x)$ and find the corresponding analytic function $f(z)$ for which $u(x, y) = \text{Re}(f(z))$. Express the result for $f(z)$ in terms of z only.

32. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.

33. How do we convert the Cauchy-Riemann equation into the corresponding polar form ? Prove the formulas for conversion in detail.

34. Show that the derived series has the same radius of convergence as the original series.

35. Determine the locus of points of z in the complex plane satisfying the equation $|z-1|+|z-2|=3$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) State and prove fundamental theorem of Algebra.

(b) Find the residues of $f(z) = \frac{z^2}{(z-1)^2(z-2)}$ at its poles.

Turn over

37. (a) State and prove Liouvillies theorem.

(b) Prove or disprove : $|\sin(z)| \leq 1$ for all complex numbers z. Justify your claim.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \frac{1}{a + b\cos\theta} d\theta, a > b > 0.$

(b) Evaluate $\int_0^{\infty} \frac{1}{x^4 + a^4} dx, a > 0.$

(2 × 13 = 26 marks)