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Name.....

Reg. No.....

**SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2026**

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Ceiling is 25.*

1. Find the Maclaurian series expansion of $f(z) = \cos z$.
2. Define the term 'removable singularity' of a complex function f .
3. What you mean by the zeros of an analytic function $f(z)$.
4. Does the series $\frac{1+i}{n^2}$ is convergent ? Justify your answer.
5. Find the radius of convergence of the series $\sum_0^\infty (-1)^n z^n / n!$.
6. Evaluate $\int_C \frac{1}{z-1}$ where C is the circle having centre at $1+i$ and radius $1/2$.
7. Does every continuous functions are differentiable ? Justify your answer.
8. Verify the Cauchy - Riemann equations for the function $f(z) = z^2 - 1$ at the point $z = 1 - i$.
9. Show that $\cosh^2 z - \sinh^2 z = 1$.
10. Write the polar form of the Cauchy - Riemann equation.

Turn over

11. Find the Principal value of $\text{Ln}(1-i)$.
12. Does the function $f(z) = \bar{z}$ is analytic? Justify your answer.
13. Does the function $f(z) = \sin(1/z)$ has an essential singularity at the origin? Justify your answer.
14. Find the residue of the function $f(z) = \frac{1}{z(z^2+1)}$ at the point $z = -i$.
15. Evaluate $\lim_{z \rightarrow 0} \frac{1-z^3}{1+z+z^2}$.

Section B

Answer any number of questions.

Each question carries 5 marks.

Overall Ceiling is 35.

16. Does the equation $\cosh 2z = 2$ has a solution? If so find the solution.
17. Define an entire function. Does the function $f(z) = \sin z$ is an entire function? Justify your answer.
18. Prove that $\lim_{n \rightarrow \infty} z_n \rightarrow 0$ if and only if $\lim_{n \rightarrow \infty} |z_n| \rightarrow 0$.
19. Show that the function $f(z) = \bar{z}$ continuous on the whole complex plane.
20. Use L'Hospital's rule find

(i) $\lim_{z \rightarrow i} \frac{1+z^6}{1+z^{10}},$

(ii) $\lim_{z \rightarrow 0} \frac{1-\cos z}{z^2}.$

21. Suppose that $f(z)$ and $\overline{f(z)}$ are analytic in a domain D . Show that $f(z)$ is constant in D .

22. Find the circle of convergence of each of the following power series :

(i) $\sum_{k=0}^{\infty} \frac{2^k}{3^k + 4^k} z^k$,

(ii) $\sum_{k=0}^{\infty} \frac{k!}{k^k} z^k$.

23. Use Cauchy's residue theorem to evaluate $\int_C \frac{1}{z^2 \sin z} dz$ where C is the circle $|z| = 1$ in the counter clockwise direction.

Section C

*Answer any two questions.
Each question carries 10 marks.*

24. Prove that $\int_0^{2\pi} \frac{d\theta}{1 + a \cos \theta} = \frac{2\pi}{\sqrt{1 - a^2}}$, $a^2 < 1$.

25. For the function $f(z) = \frac{1}{z(z-1)(z-2)}$, find the Laurent series expansion in

(i) the domain $|z| < 1$;

(ii) the domain $1 < |z| < 2$; and

(iii) the domain $|z| > 2$.

26. Show that $u = x^2 - y^2 - x + 1$ is harmonic. Find a harmonic conjugate v of u such that $u + iv$ is analytic.

27. State and Prove Cauchy's Integral formula.

(2 × 10 = 20 marks)