D 130226 **(Pages : 3)** **Name.....**

Reg. No.....

FIFTH SEMESTER (CBCSS—U.G.) DEGREE EXAMINATION
NOVEMBER 2025

Mathematics

MTS 5B 05—ABSTRACT ALGEBRA

(2020 Syllabus)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Ceiling is 25.

1. Define Euler's totient function ϕ hence find $\phi(10)$.
2. Check whether the relation on R defined by $a \sim b$ if $a \leq b$, where $a, b \in R$ is an equivalence relation.
3. Write the permutation :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 2 & 7 & 6 & 3 & 8 & 1 & 4 \end{pmatrix}$$

in S_8 as disjoint product of cycles and as a product of transpositions.

4. Show that in a group G and $a, b \in G$, each of the equations $ax = b$ and $xa = b$ has a unique solution.
5. Write the addition table for Z_8.
6. Let G be a group and $a \in G$. Then show that $\langle a \rangle$ is a sub-group of G.
7. Show that any group of prime order is cyclic.
8. Let $\phi : G_1 \to G_2$ be an isomorphism of groups. If a has order n in G_1, then show that $\phi(a)$ also has order n in G_2.

Turn over

9. Show that the set of all even permutations of S_n is a sub-group of S_n.

10. Give the sub-group diagram of Z_{60}.

11. Find the order of the permutation (1, 2, 3) (2, 4, 3, 5) (1, 3, 2) in S_5.

12. Let $\phi : G_1 \to G_2$ be a group isomorphism. Show that $\left(\phi(a)\right)^{-1} = \phi\left(a^{-1}\right)$ for all $a \in G$.

13. Show that sub-groups of index two are normal.

14. Let G be a group. Prove that Aut (G) is a group under composition of functions.

15. Define units in a ring. What are the units in Z ?

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Ceiling is 35.

16. Let $S = R - \{-1\}$. Define * on S by $a * b = a + b + ab$. Show that (S, *) is a group.

17. Prove that every permutation in S_n can be written as a product of disjoint cycles.

18. Let G be a group, and let H and K be sub-groups of G. If $h^{-1} kh \in K$ for all $h \in H$ and $k \in K$, then HK is a subgroup of G.

19. Let G be an infinite cyclic group. Show that $G \cong Z$.

20. Let G be a group, and let $a, b \in G$ be elements such that $ab = ba$. If the orders of a and b are relatively prime, then $o(ab) = o(a)\, o(b)$.

21. Show that any finite integral domain is a field.

22. Let $\phi : G_1 \to G_2$ be a group homomorphism. If H_1 is a sub-group of G_1, then show that $\phi(H_1)$ is a sub-group of G_2.

23. State and prove First Isomorphism Theorem.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. Let S be any set and σ, τ are disjoint cycles in Sym (S). Show that $\sigma\tau = \tau\sigma$.

25. Show that every group is isomorphic to a group of permutations.

26. Let H be a sub-group of the finite group G. Show that the order of H is a divisor of order of G.

27. State and prove Second Isomorphism Theorem.