D 130228 **(Pages : 4)** **Name.......................................**

Reg. No...................................

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2025

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2020 Syllabus)

Time : Two Hours Maximum : 60 Marks

Section A

Not more than 20 marks can be earned from this unit.
Each question carries 2 marks.

1. Suppose that $f \in C[a, b]$ and $f(a) \cdot f(b) < 0$. Prove that the Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero p of f with $|p_n - p| \le \frac{b-a}{2^n}$, when $n \ge 1$.

2. Find a root of $f(x) = x^2 - 3x + 1 = 0$, by fixed point iteration method.

3. Apply Newton's method to solve the algebraic equation $f(x) = x^3 + x - 1 = 0$ correct to 6 decimal places. (Start with $p_0 = 1$).

4. Determine the linear Lagrange interpolating polynomial that passes through the points $(2, 4)$ and $(5, 1)$.

5. Complete the divided difference table for the data given in the following table :

x	:	0	1	2	4
$f(x)$	:	1	1	2	5

6. Write the Second Derivative Midpoint Formula to approximate second derivative.

7. Describe briefly : The $(n+1)$-point closed Newton-Cotes formula.

8. Describe briefly : composite Simpson's rule.

Turn over

9. Describe briefly : Adams-Bashforth Two-Step Explicit Method

10. Give the formula for local truncation error at the i^{th} step of Euler's method.

11. Describe Runge Kutta method of Order Two.

12. Approximate the integral $\int_1^{1.5} x^2 \ln x \, dx$ using Gaussian quadrature with $n = 2$.

Section B

Not more than 30 *marks can be earned from this unit.*
Each question carries 5 *marks.*

13. Find ln 9.2 with $n = 3$, using Lagrange's interpolation formula with the given table :

x	:	9.0	9.5	10.0	11.0
$\ln x$	:	2.197 22	2.251 29	2.302 59	2.397 90

14. Using method of false position, find a real root of the equation,

$f(x) = x^3 + x - 1 = 0$, near $x = 1$.

15. Use the trapezoidal rule to estimate

$$\int_1^2 \frac{1}{x} \, dx.$$

16. Find an approximate value of $\log_e 5$ by calculating $\int_0^5 \frac{dx}{4x+5}$, by Simpson's 1/3 rule of integration.

17. If Taylor's method of order n is used to approximate the solution to

$y'(t) = f(t, y(t)), a \le t \le b, y(a) = \alpha$

with step size h and if $y \in C^{n+1}[a, b]$, then prove that the local truncation error is $O(h^n)$.

18. Use Taylor's method of order two to approximate the solutions for the following initial-value problem :

$$y' = te^{3t} - 2y, 0 \le t \le 1, y(0) = 0, \text{ with } h = 0.5.$$

19. Using modified Euler's method, determine the value of y when x = 0.05 given that

$$y' = t^2 + y \,;\, y(0) = 1. (\text{Take } h = 0.05)$$

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

20. (a) Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three to approximate $f(8.4)$ if the following hold :

$$f(8.1) = 16.94410, f(8.3) = 17.56492, f(8.6) = 18.50515, f(8.7) = 18.82091$$

(b) For the following table of values, estimate $f(7.5)$, using Newton's backward difference interpolation formula.

x	f	∇f	$\nabla^2 f$	$\nabla^3 f$	$\nabla^4 f$
1	1				
2	8	7	12	6	
3	27	19	18	6	0
4	64	37	24	6	0
5	125	61	30	6	0
6	216	91	36	6	0
7	343	127	42		
8	512	169			

Turn over

21. (a) Given the initial value problem $y' = x + y$, $y(0) = 0$. Find the value of y approximately for $x = 1$ by Euler method in five steps. Compare the result with the exact value.

(b) Use Runge-Kutta method with $h = 0.1$ to find $y(0.2)$ given $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 0$.

(1 × 10 = 10 marks)

658430