

## FIRST SEMESTER M.A. DEGREE EXAMINATION, NOVEMBER 2018

(CUCSS—PG)

Economics

ECO 1C 04—QUANTITATIVE METHODS FOR ECONOMIC ANALYSIS—I

[2015 Syllabus Year]

Time : Three Hours

Maximum : 36 Weightage

## Part A

Answer all questions. Weightage 1 for each question.

1. Evaluate  $\begin{vmatrix} -1 & 2 & -3 \\ 2 & -3 & -1 \\ -3 & -1 & 2 \end{vmatrix}$ .

2. Find the rank of  $\begin{bmatrix} 4 & 1 & 2 \\ -3 & 2 & 4 \\ 8 & -1 & -2 \end{bmatrix}$ .

3. Show that the characteristic equation of the square matrix  $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$  is  $\lambda^3 - 3\lambda^2 - 7\lambda - 11 = 0$ .

4. Let an exponential function be  $y = a^x$ , find  $\frac{dy}{dx}$ .

5. If the total cost of making  $x$  litres of an acid is  $T = -30 + 80x^{\frac{1}{2}}$  rupees, Find the number of units at which the marginal cost is Rs. 1.25.

6. Find the total derivative of  $u = x^2 y^3 + x^3 y^2$ .

7. The cost for a monopolist firm producing  $x$  mobile phones per week is given to be  $4x^2 - 80x + 500$  rupees. To have minimum cost, how many units should be produced per week?

8. Integrate  $\frac{3x^3 - 5x^2 + 6x - 8}{x}$  with respect to  $x$ .

9. Find  $\int \frac{x-5}{x^2 - 10x + 11} dx$ .

10. Give the axiomatic definition of probability.

11. State the addition theorem of probability.

12. If  $E(x) = 2.5$ , find  $E(3X + 7)$ .

(12 × 1 = 12 weightage)

### Part B

Answer any **eight** questions. Weightage 2 for each question.

13. If  $A = \begin{bmatrix} 1 & 1 & -1 \\ -2 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix}$ , show that  $\frac{1}{2}(A - A^T)$  is skew-symmetric.

14. Given that  $A = \begin{bmatrix} 2 & 3 & 6 \\ 3 & -6 & 2 \\ -6 & -2 & 3 \end{bmatrix}$ , find  $A^T A$ . Hence or otherwise evaluate  $A^{-1}$ . What is the peculiarity of the matrix A?

15. A manufacturer produces three products A, B, C which are sold in Delhi and Calcutta. Annual sales of these products are given below :

	Products		
	A	B	C
Delhi	5000	7500	15000
Calcutta	9000	12000	8700

If the sale price of the products A, B, C per unit be Rs. 20, Rs. 30, Rs. 40 respectively, calculate the total revenue in each, centre by using matrices.

16. Using the function  $f(x, y) = x^2 + y^2 - 2xy + 8x + 9y + 3$ , show that  $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ .

17. Given the production function  $q = 165 - 3p - 2p^2$ , find the elasticity of demand at the price  $p = 5$ .

18. Given the production function  $V = (\beta k^{-\rho} + \alpha L^{-\rho})^{-\frac{1}{\rho}}$ , when V is the output, k is capital, L is labour, and  $\alpha, \beta, \rho$  are constants. Find  $dV$ .

19. Find the maximum and minimum values of the function  $y = 2x^3 - 3x^2 - 36x + 12$ .

20. Find the minimum of  $u = 2x^2 - 6y^2$  under the condition that  $x + 2y = 4$ . What is the value of  $u$  ?
21. Evaluate  $\int x^3 \log x dx$ .
22. A committee of 5 is to be formed from a group of 8 boys and girls. Find the probability that the committee consists of (i) 3 boys and 2 girls ; (ii) at least one girl.
23. If it rains, a taxi driver can earn Rs. 1,000 per day. If it is fair, he can earn only Rs. 800 per day. If the probability of rain is 0.4, what is his expectation ?
24. A variable  $X$  has the probability density function  $f(x) = 6x(1-x)$ ;  $0 \leq x \leq 1$ . Find the first two moments about mean. Also find mean and variance.

(8 × 2 = 16 weightage)

**Part C***Answer any two questions. Weightage 4 for each question.*

25. Solve the following using matrix method :

$$\begin{aligned}x + y + z &= 9 \\2x + 5y + 7z &= 52 \\2x + y - z &= 0\end{aligned}$$

26. (a) If  $y = e^x \log x$  show that  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - x(x+1)y = 2xe^x$ .
- (b) Find the total derivative of  $u$  with respect to  $t$  if  $u = x^2 + y^2$ ;  $x = t^3 + 3$ .
27. (a) The net revenue of a monopolist producing  $x$  tons of chocolate is given by  $\frac{x(75-x)}{3} - \left(\frac{x^2}{25} + 3x + 100\right)$ . Find the level of output at which the net revenue will be maximum.
- (b) Given  $y = (2-x)^2 + (3-x)^2 + (4-x)^2$ , show that  $y$  is a minimum when  $x$  is equal to the arithmetic mean of 2, 3 and 4.
28. A factory produces a certain type of outputs by three types of machines. The respective daily production figures are Machine I : 3000 units, Machine II : 2500 units, Machine III : 4500 units. Past experience shows that 1% of the output produced by machine I is defective. The corresponding fractions of defectives for the other two machines are respectively 1.2% and 2%. An item is drawn at random from the days production run and is found to be defective. What is the probability that it comes from the output of (a) Machine I ; (b) Machine II ; and (c) Machine III ?

(2 × 4 = 8 weightage)