

D 140687

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Name.....

Reg. No.....

**SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE
EXAMINATION, APRIL 2026**

Mathematics

MTH2C06—ALGEBRA—II

(2019 Admission onwards)

Time : Three Hours

Maximum Weightage : 30

Part A*Answer all questions.**Each question carries a weightage 1.*

1. Verify whether $6\mathbb{Z}$ is a maximal ideal of the ring $\mathbb{Z}$ of integers.
2. Find the characteristic of the field $\mathbb{Z}_5$ of the integers mod 5.
3. Verify whether $\sqrt{\pi}$ is algebraic over the field $\mathbb{Q}(\pi)$.
4. Find all automorphisms of the field $\mathbb{Q}(\sqrt{2})$.
5. Let σ be the automorphism of the field $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ such that $\sigma(\sqrt{2}) = \sqrt{2}$ and $\sigma(\sqrt{3}) = -\sqrt{3}$. Find the fixed field of σ .
6. Find the splitting field of the polynomial $x^3 - 2$ over $\mathbb{Q}$.
7. Let $K = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Find the subgroup of $G(K/\mathbb{Q})$ that correspond to $E = \mathbb{Q}(\sqrt{3})$ in the Galois correspondence.
8. Find the splitting field of the polynomial $x^5 - 1$ over $\mathbb{Q}$.

(8 × 1 = 8 weightage)

Part B*Answer any two questions from each module.**Each question carries a weightage 2.***Module I**

9. Let F be a field of characteristic zero. Show that F contains a subfield isomorphic to the field $\mathbb{Q}$ of rationals.

Turn over

10. Let $p(x)$ be an irreducible polynomial in $F[x]$ where F is a field. Show that the ideal generated by $p(x)$ is a maximal ideal in $F[x]$.
11. Find the algebraic closure of the rationals $\mathbb{Q}$ in the field $\mathbb{C}$ of complex numbers.

Module II

12. Let E, E' be field of order p^n where p is a prime and n is a positive integer. Show that E and E' are isomorphic fields.
13. Let $F \leq E \leq \bar{F}$ be fields where $\bar{F}$ is the algebraic closure of F . Let E be a splitting field over F . Show that if σ is an automorphism of $\bar{F}$ leaving F fixed then $\sigma(a) \in E$ for all $a \in E$.
14. Let p be a prime and $E = \mathbb{Z}_p(y)$ where y is an indeterminate. Let $t = y^p$ and $F = \mathbb{Z}_p(t) \leq E$. Show that E is an algebraic extension of F .

Module III

15. Describe all elements of the Galois group $G(K/\mathbb{Q})$ where $K = \mathbb{Q}(\sqrt{2}, \sqrt{3})$.
 16. Show that the 8th cyclotomic polynomial $\phi_8(x)$ is $x^4 + 1$.
 17. Show that regular 18-gon is not constructible by ruler and compass.
- (6 × 2 = 12 weightage)

Part C

*Answer any **two** questions.
Each question carries a weightage 5.*

18. (a) Define maximal ideal in a ring.
(b) Let R be a commutative ring with unity and M be an ideal of R . Show that M is a maximal ideal of R if and only if R/M is a field.
19. (a) Show that the set of all constructible real numbers forms a subfield of the field of reals.
(b) Show that doubling the cube is an impossible geometric construction.
20. Let F be a finite field of characteristic p . Let $\sigma_p : F \rightarrow F$ be defined by

$$\sigma_p(a) = a^p \text{ for all } a \in F.$$

Show that :

- (a) σ_p is an automorphism of F .
- (b) The fixed field of σ_p is isomorphic to $\mathbb{Z}_p$.

21. (a) Define normal extension of a field.
- (b) Let K be a finite normal extension of a field F and E be an extension of F such that $E \leq K$. Show that :
- (i) K is a finite normal extension of E .
 - (ii) $G(K/E)$ is a subgroup of $G(K/F)$.

(2 × 5 = 10 weightage)