

D 93241

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Name.....

Reg. No.....

**FIRST SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION
NOVEMBER 2020**

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA—I

(2016 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer all questions.

Each question carries weightage 1.

1. Verify whether $\phi(x, y) = (x + 2, y + 3)$ is an isometry of the Euclidean plane \mathbb{R}^2 .
2. Give two non-isomorphic groups of order 10.
3. Find the order of $(3, 10, 9)$ in the group $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{15}$.
4. Show that if G is non-abelian, then the factor $G/Z(G)$ is not cyclic.
5. Find the ascending central series of S_3 .
6. Verify whether the series $\{0\} < 10\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 25\mathbb{Z} < \mathbb{Z}$ are isomorphic.
7. Give an example to show that a factor ring of an integral domain may be a field.
8. Derive class equation.
9. For a prime p , prove that every group G of order p^2 is abelian.
10. Prove that a non-zero polynomial $f(x) \in F[x]$ of degree n can have almost n zeros in a field F .
11. For $n \in \mathbb{N}$, prove that $\mathbb{Z}/n\mathbb{Z}$ is isomorphic to \mathbb{Z}_n .
12. Prove that no group of order 36 is simple.

Turn over

13. Let H and K be normal subgroups of a group G . Give an example showing that we may have H isomorphic to K but G/H is not isomorphic to G/K .
14. Prove that $f(x) = x^4 - 2x^2 + 8x + 1$ is irreducible over \mathbb{Q} .

(14 × 1 = 14 weightage)

Part B

Answer any **seven** questions.

Each question has weightage 2.

15. Find all abelian groups, upto isomorphism, of order 360.
16. Let H be a normal subgroup of G . Prove that the cosets of H form a group G/H under the binary operation $(aH)(bH) := (ab)H$.
17. Find isomorphic refinements of the series $\{0\} < 8\mathbb{Z} < 4\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 9\mathbb{Z} < \mathbb{Z}$.
18. Let p be a prime. Let G be a finite group and let p divides $|G|$. Prove that G has a subgroup of order p .
19. What is meant by a free group?
20. Show that the presentation $(a, b : a^3 = 1, b^2 = 1, ba = a^2b)$ gives a group isomorphic to S_3 .
21. Let F be a field and let $f(x) \in F[x]$. Suppose $f(x)$ is of degree 2 or 3. Prove that $f(x)$ is irreducible over F if and only if it has a zero in F .
22. Let F be the ring of all functions mapping \mathbb{R} into \mathbb{R} , and let C be the subring of F consisting of all the constant functions in F . Is C an ideal in F ? why?
23. Let G be a group. If H is any subgroup of G and N is a normal subgroup of G . Prove that HN is a subgroup of G .
24. Let H and K be normal subgroups of a group G with $K \leq H$. Show that G/H is isomorphic to $(G/K)/(H/K)$.

(7 × 2 = 14 weightage)

Part C

Answer any **two** questions.

Each question has weightage 4.

25. Prove that every finite group of isometries of the plane is isomorphic to either \mathbb{Z}_n or to a dihedral group D_n for some positive integer n .

26. Let X be a G -set. For $x, y \in X$ let $x \sim y$ if $gx = y$ for some $g \in G$. Show that :

(a) \sim is an equivalence relation on X .

(b) If $[x]$ is the equivalence class containing x then

$$|[x]| = (G : G_x) \text{ where } G_x = \{g \in G : gx = x\}.$$

27. State and prove first Sylow theorem.

28. (a) State Eisenstein Criterion for irreducibility of polynomials in $\mathbb{Q}[x]$.

(b) Prove that the polynomial $\phi_p(x) = \frac{x^p - 1}{x - 1}$ is irreducible over \mathbb{Q} .

(2 × 4 = 8 weightage)