

Q.P Code <b>D 123237</b>	Total Pages <b>3</b>	Name <b>604759</b>
		Register No.
<b>SECOND SEMESTER (CUFYUGP) DEGREE EXAMINATION, APRIL 2025</b>		
<b>MATHEMATICS</b>		
<b>MAT2MN102 Calculus and Matrix Algebra</b>		
<b>2024 Admission Onwards</b>		
<b>Maximum Time :2 Hours</b>		<b>Maximum Marks :70</b>

### Section A

All Question can be answered. Each Question carries 3 marks (Ceiling : 24 Marks)

1	Find $\int \frac{\sin x}{\cos^2 x} dx$
2	Find $\int \sin^3 x dx$
3	Find the average value of the function $f(x) = \frac{1}{x}$ over the interval $[1, e]$
4	Evaluate $\int_0^4 3x\sqrt{25-x^2} dx$
5	Find $\int xe^x dx$
6	Describe the level surfaces of $f(x, y, z) = z^2 - x^2 - y^2$
7	Explain the domain of $f(x, y) = \ln xy$
8	For $n \times m$ matrices $\mathbf{A}$ and $\mathbf{B}$ , show that $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
9	True or False: "For any $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$ , we get $(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$ ". Explain your answer
10	Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} -8 & 1 \\ 16 & 0 \end{pmatrix}$

## Section B

All Question can be answered. Each Question carries 6 marks (Ceiling : 36 Marks))

11	Evaluate the integral $\int_1^7 f(x) dx$ , if $f(x) = \begin{cases} x^2 & 1 \leq x < 3 \\ 2x + 2 & 3 \leq x < 5 \\ x - 1 & 5 \leq x \leq 7 \end{cases}$
12	Using definite integral, find the area of the region below the curve $y = x - x^2$ and above the $x$ - axis.
13	Find the area of the region enclosed by the curves $y = x^3 - 4x, y = 0, x = 0, x = 2$
14	Evaluate $\int e^x \cos x dx$
15	Determine whether the limit exists. If so, find its value. $\lim_{(x,y) \rightarrow (0,0)} \frac{\cos xy}{x^2 + y^2}$
16	If, $f(x, y, z) = \tan^{-1} \left( \frac{1}{xy^2z^3} \right)$ , then find $f_x, f_y$ and $f_z$
17	Solve the following system of linear equations $\begin{aligned} x_1 + 2x_2 + 2x_3 &= 2 \\ x_1 + x_2 + x_3 &= 0 \\ x_1 - 3x_2 - x_3 &= 0 \end{aligned}$
18	Construct an orthogonal matrix from the eigenvectors of the symmetric matrix $\begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$

## Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks))

19	Evaluate the integral $\int_0^3 \sqrt{6x - x^2} dx$ by completing the square and applying appropriate formulas from geometry.
20	Let $f(x, y, z) = \sqrt{x^2 - 2y^2 + 4z^2}$ , find <ol style="list-style-type: none"><li data-bbox="287 627 462 739">1. <math>\left. \frac{\partial f}{\partial x} \right _{(2,1,1)}</math></li><li data-bbox="287 739 462 851">2. <math>\left. \frac{\partial f}{\partial y} \right _{(1,2,3)}</math></li><li data-bbox="287 851 462 963">3. <math>\left. \frac{\partial f}{\partial z} \right _{(2,1,3)}</math></li></ol>