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C 4748 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2021

(CBCSS)

Mathematics

MT 2C 06—ALGEBRA—II

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Show that a commutative ring with unity is a field iff it has no proper non-trivial ideals.
2. Show that $\sqrt{1+\sqrt{3}}$ is algebraic over Q.
3. Show that doubling the cube is impossible.
4. What is the order of $G\left(Q\left(\sqrt[3]{2}\right)/Q\right)$?
5. Prove that if E is an algebraic extension of a perfect field F, then E is perfect.
6. Show that the Galois group of the p^{th} cyclotomic extension of Q for a prime p is cyclic of order $p-1$.
7. Show that the regular 18-gon is not constructible.
8. Show that the polynomial x^5-1 is solvable by radicals over Q.

($8 \times 1 = 8$ weightage)

Turn over

Part B

Answer any **two** *questions from each of the following 3 units.*
Each question carries 2 weightage.

UNIT I

9. Let E be a simple extension $F(\alpha)$ of a field F, and let α be algebraic over F. Let the degree of irr (α, F) be $n \geq 1$. Show that every element β of $E = F(\alpha)$ can be uniquely expressed in the form $\beta = b_0 + b_1 \alpha + ... + b_{n-1} \alpha^{n-1}$, where the b_i are in F.

10. Show that $Q\left(2^{1/2}, 2^{1/3}\right) = Q\left(2^{1/6}\right)$.

11. Show that a field F is algebraically closed iff every non-constant polynomial in $F[x]$ factors in $F[x]$ into linear factors.

UNIT II

12. Find all the primitive 18th roots of unity in $GF(19)$.

13. Let F be a finite field of characteristic p. Show that the map $\sigma_p : F \to F$ defined by $\sigma_p(a) = a^p$ for $a \in F$ is an automorphism.

14. Show that if K is a finite extension of E and E is a finite extension of F, then K is separable over F iff K is separable over E and E is separable over F.

UNIT III

15. State the Main Theorem of Galois Theory.

16. Find $\phi_{12}(x)$ *in* $Q[x]$.

17. Let F be a field of characteristic zero and F contains all the n^{th} roots of unity. Show that if K is the splitting field of $x^n - a$ over F for some $a \in F$, then $G(K|F)$ is a soluble group.

(6 × 2 = 12 weightage)

Part C

*Answer any **two** questions.*
Each question carries 5 weightage.

18. (a) Let F be a field. Show that an ideal $< p(x) > \neq \{0\}$ of $F[x]$ is maximal iff $p(x)$ is irreducible over F.

    (b) Show that $\dfrac{z_5[x]}{< x^3 + 3x + 2 >}$ is a field.

19. (a) Show that if E is finite extension field of a field F, and K is a finite extension field of E, then K is a finite extension of F, and $[K:F] = [K:E][E:F]$.

    (b) Show that if E is a finite extension of F, then $\{E:F\}$ divides $[E:F]$.

20. State and prove the theorem of the conjugation isomorphisms.

21. Let K be the splitting field of $x^4 + 1$ over Q :

    (i) Describe the group $G(K|Q)$; and

    (ii) Give the group and field diagrams for K over Q.

(2 × 5 = 10 weightage)