D 131313 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

(CBCSS)

Mathematics

MTH 1C 01—ALGEBRA—I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Find the order of $(2, 3)$ in the group $\mathbb{Z}_6 \times \mathbb{Z}_{15}$.

2. Give two non-isomorphic groups of order 18.

3. Let X be a G-set, $Y \subseteq X$ and $G_Y = \{g \in G \mid gy = y, \forall y \in Y\}$. Prove that G_Y is a subgroup of G.

4. Use the First Isomorphism Theorem to prove that $\mathbb{Z} \times \mathbb{Z} / \{0\} \times \mathbb{Z} \simeq \mathbb{Z}$.

5. Define composition series. Give example.

6. Find the number of Sylow 5-Subgroups of a group of order 20.

7. Give a presentation of S_3 involving two generators.

8. Find the product of $1 + 2i + 3j$ and $1 - 2i - 2k$ in the ring of quaternions.

(8 × 1 = 8 weightage)

Turn over

Section B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

MODULE I

9. Prove that a factor group of a cyclic group is cyclic.

10. Let M be a normal subgroup of a group G. Prove that if G/M is simple, then M is a maximal normal subgroup of G.

11. If m is a square free, integer, then prove that every abelian group of order m is cyclic.

MODULE II

12. If G has a composition series and N is a proper normal subgroup of G, then prove that there exists a composition series containing N.

13. Prove that no group of order 30 is simple.

14. If G is a group of order p^n and X is a finite G -set. Then show that $|X| \equiv |X_G| (\text{mod } p)$.

MODULE III

15. Show that $(x, y : y^2 x = y, yx^2y = x)$ is a presentation of the trivial group of one element.

16. Prove that the polynomial $x^2 - 2$ has no zeroes in the set of rational numbers.

17. Prove that $\mathbb{Z}/10\mathbb{Z} \simeq \mathbb{Z}_{10}$.

(6 × 2 = 12 weightage)

Section C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. a) State the Fundamental Theorem of Finitely Generated Abelian Groups.

    b) Let X be a G -set and $x \in X$. Prove that $|Gx| = (G, G_x)$, where Gx is the orbit and G_x is the isotropy subgroup of x.

19. State and prove the Third Sylow Theorem. Verify the Third Sylow Theorem for S_3.

20. Let H be a subgroup of a group G and N be a normal subgroup of G . Prove that $HN/N \simeq H/H \cap N$.

21. a) State Division algorithm for $F[x]$, where F is a field.

    b) Show that the polynomial $x^4 - 2x^2 + 8x + 1$ is irreducible over $\mathbb{Q}$.

(2 × 5 = 10 weightage)