

D 31819

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Name.....

Reg. No.....

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019 Admission Onwards)

Time : Two Hours

Maximum : 60 Marks

Section A

Answer any number of questions.
Maximum 20 marks.

1. Find the derivative of the vector function $\vec{r}(t) = \sin t \hat{i} - e^{-t} \hat{j} + (3t^3 - 4) \hat{k}$.
2. If $z = 4x^3y^2 - 6x^2 + y^2 + 5$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
3. If $f(x, y) = e^{xy}$, find $\nabla f(x, y)$.
4. Find the level curve of $f(x, y) = y^2 - x^2$ passing through the point $(-1, 2)$.
5. Find $\text{div } \vec{F}$ for $\vec{F} = (x^2y^3 - z^4) \hat{i} + 4x^5y^2z \hat{j} - y^4z^6 \hat{k}$.
6. Evaluate $\int_{1-1}^{3 \ 1} \int (2x - 4) dx$.
7. State Stoke's theorem.
8. Find the Jacobian of $x = r \cos \theta$, $y = r \sin \theta$.
9. Express $1 + i$ in polar form.
10. Evaluate $\lim_{z \rightarrow i} \frac{z^4 - 1}{z - i}$.

Turn over

11. Evaluate $\oint_C \frac{e^z}{z-3} dz$ where C is $|z|=1$.

12. Evaluate $\oint_C \bar{z} dz$ where C is $x=t, y=t^2, 0 \leq t \leq 1$.

Section B

Answer any number of questions.
Maximum 30 marks.

13. Use chain rule to find $\frac{\partial z}{\partial u}$ at $(\pi, 1)$ for $z = x^2 - y^2 \tan x$, where $x = \frac{u}{v}, y = uv$.

14. Find an equation of the tangent plane to the graph of $z = \frac{x^2}{2} + \frac{y^2}{2} + 4$ at $(1, -1, 5)$.

15. Show that $\int_C (y^2 - 6xy + 6) dx + (2xy - 3x^2) dy$ is independent of any path C between $(-1, 0)$ and

$(3, 4)$. Hence evaluate $\int_{(-1,0)}^{(3,4)} (y^2 - 6xy + 6) dx + (2xy - 3x^2) dy$.

16. Change the order of integration and hence evaluate $\int_0^4 \int_y^4 \frac{x}{x^2 + y^2} dx dy$.

17. Show that $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic. Find the harmonic conjugate of u .

18. Evaluate $\int_C z^2 dz$ where C is the line $x = 2y$ from $z = 0$ to $z = 2 + i$.

19. Evaluate $\oint_C \frac{z+1}{z^4 + 4z^3} dz$ where C is $|z| = 1$.

Section C

Answer any **one** question.

Maximum 10 marks.

20. Use Green's theorem to evaluate $\oint_C (x^5 + 3y) dx + (2x - e^{y^3}) dy$ where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

21. Find the volume bounded by the cylinder $x^2 + y^2 = 4$, the plane $y + z = 3$ and $z = 0$.

(1 × 10 = 10 marks)