C 22093 (Pages : 3) Name.....................................

Reg. No.................................

SECOND SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 2C 01—MATHEMATICS—2

(2021 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Find the inverse of $f(x)=\dfrac{2x-3}{5x-7}$, where the domain of f excludes $x=\dfrac{7}{5}$.

2. Find the Cartesian form of the polar equation $r = \sin 2\theta$.

3. Express the number $\coth^{-1}(5/4)$ in terms of natural logarithms.

4. Prove that $\tanh^2 x + \operatorname{sech}^2 x = 1$.

5. Show that the series $1+\dfrac{1}{2}+\dfrac{1}{2^2}+\dots$ converges and also find its sum.

6. Find the norm of the vector (3, 4, 0, 1, – 1). Also normalize the vector.

7. Determine the radius of convergence of $\sum\limits_{k=0}^{\infty}\dfrac{k^5}{(k+1)!}x^k$.

8. Find a basis and then give the dimension of solution space of.

9. Find the inner product of the vectors $\mathbf{a}=\langle 1,2,3\rangle$ and $\mathbf{b}=\langle 0,-2,1\rangle$ in R^3. Are the vectors orthogonal ?

10. Show that $A=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$ is an orthogonal matrix.

Turn over

11. If $A = \begin{bmatrix} 10 & 3 \\ 4 & 6 \end{bmatrix}$ find A^3 using Cayley Hamilton theorem.

12. Find the inverse of the 2 × 2 matrix $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Find the length of the graph of $f(x) = (x-1)^{3/2} + 2$ on [1, 2].

14. Diagonalize the matrix $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$.

15. Find the length of the perimeter of the cardioid $r = a(1 + \cos\theta)$.

16. Find an approximation value of $\int_0^1 x^2 dx$ by Simpson's rule with $n = 10$.

17. Expand $\log x$ in ascending powers of $x - 1$ as for the term containing $(x - 1)^4$.

18. $B_1 = \{u_1, u_2, u_3\}$, where $u_1 = \langle 2, -1, 1 \rangle, u_2 = \langle 1, 5, 1 \rangle, u_3 = \langle 0, 1, 2 \rangle$, is a basis for $\mathbb{R}^2$. Transform it into an orthonormal basis $B_2 = \{w_1, w_2, w_3\}$.

19. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing it to the echelon form.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 marks.

20. (a) Find the area of the region shared by the circles $r = 1$ and $r = 2 \sin \theta$.

    (b) Show that the set $B = \{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ is a basis for R^3.

21. (a) Using Gauss-Jordan elimination method, solve the system of equations :

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + y - 2z &= 1 \\ 4x - 3y - z &= 3 \\ 2x + 4y + 2z &= 4. \end{aligned}$$

    (b) Find the eigen values of $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$.

(1 × 11 = 11 marks)