C 40606 **(Pages : 4)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS—UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Short Answer Type Questions.
Ceiling 25 Marks.

1. Find the solution of the differential equation $\frac{dp}{dt} = 0.5p - 450$.

2. Solve the differential equation $\left(4+t^2\right)\frac{dy}{dt} + 2ty = 4t$.

3. State Existence and Uniqueness Theorem for First-Order Linear Differential Equations.

4. Solve the initial value problem $y' = y^2, \quad y(0) = 1$.

5. Find the general solution of $y'' + 5y' + 6y = 0$.

6. If y_1 and y_2 are two solutions of the differential equation, $y'' + p(x)y' + q(x)y = 0$.

   Show that $c_1y_1 + c_2y_2$ is also a solution for any values of the constants c_1 and c_2.

7. Let $y_1 = e^t \sin t, y_2 = e^t \cos t$. Find the Wronskian $W[y_1, y_2]$.

8. Solve the differential equation $y'' - 2y' - 3y = 3e^{2t}$.

9. Find the Laplace transform of e^{at}.

Turn over

10. Find $\mathcal{L}^{-1}\left(\frac{s}{s^2 - a^2}\right)$ for $s > | a |$.

11. If $F(s) = \mathcal{L}(f(t))$ exists for $s > a \geq 0$, and if c is a constant, Show that

$\mathcal{L}\left(e^{ct} f(t)\right) = F(s - c), \quad s > a + c.$

12. Solve the boundary value problem $y'' + y = 0, \; y(0) = 1, y(\pi) = a,$

where a is a given number.

13. Find the fundamental period of the function $\sin(5x)$.

14. Define an odd function. Prove that if $f(x)$ is an odd function then

$\int_{-L}^{L} f(x)\, dx = 0.$

15. Verify that the method of separation of variables may be used to solve the equation $xu_{xx} + u_t = 0$.

(2 Marks each)

Section B

Paragraph / Problem Type Questions.
Ceiling 35 Marks.

16. Show that the equation

$\frac{dy}{dx} = \frac{x^2}{1 - y^2}.$

is separable, and then find an equation for its integral curves.

17. Solve the differential equation

$\left(y \cos x + 2xe^y\right) + \left(\sin x + x^2 e^y - 1\right) y' = 0.$

18. Given that $y_1(t) = t^{-1}$ is a solution of

$$2t^2y'' + 3ty' - y = 0, t > 0,$$

find a fundamental set of solutions.

19. Find the general solution of the differential equation $y'' + y = \tan t$ on $0 < t < \pi/2$.

20. Find the Laplace transform of the following function $f(t) = \int_0^t (t-\tau)^2 \cos(2\tau)\, d\tau$.

21. Find the inverse Laplace transform of the following function using the convolution theorem

$$F(s) = \frac{1}{(s+1)^2(s^2+4)}.$$

22. Determine the co-efficients in the Fourier series of the function

$$f(x) = \begin{cases} -x, & -2 \le x \le 0, \\ x, & 0 \le x \le 2 \end{cases}$$

with $f(x+4) = f(x)$.

23. Find the displacement $u(x,t)$ of the vibrating string of length $L = 30$ that satisfies the wave equation

$$4u_{xx} = u_{tt}, \quad 0 < x < 30, t > 0.$$

Assume that the ends of the string are fixed and that the string is set in motion with no initial velocity from the initial position

$$u(x,0) = f(x) = \begin{cases} x/10, & 0 \le x \le 10, \\ (30-x)/20, & 10 < x \le 30 \end{cases}.$$

(5 marks each)

Turn over

Section C (Essay Type Questions)

two *out of four.*

24. (a) Find the general solution of the differential equation $\frac{dy}{dt} - 2y = 4 - t$ by the method of integrating factors.

(b) Find the value of b for which the following equation is exact, and then solve it using that value of b

$$\left(ye^{2xy} + x\right) + bxe^{2xy}y' = 0.$$

25. Find a series solution in powers of x of Airy's equation

$$y'' - xy = 0, \quad -\infty < x < \infty.$$

26. Use the Laplace transform and solve the following initial value problem

$$y'' + 3y' + 2y = 0; \quad y(0) = 1, y'(0) = 0.$$

27. Find the Fourier series of the following periodic function $f(x)$ of period $p = 2L$ defined by

$$f(x) = 3x^2 \quad -1 < x < 1.$$

(10 marks each)