

C 20646

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Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS-UG)

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A

*Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

1. Define holomorphic function in a domain D . And give an example for an entire function.
2. Prove or disprove : if f is differentiable a point z_0 , then f is continuous at that point.
3. Define harmonic function with example.
4. Prove that $\sin^2 z + \cos^2 z = 1$.
5. State ML inequality.
6. Define the path independence for a contour integral.
7. State maximum modulus theorem.
8. Prove that $\int_a^b f(t) dt = -\int_b^a f(t) dt$.
9. Prove or disprove if $\lim_{n \rightarrow \infty} z_n = 0$, then $\sum_{k=1}^{\infty} z_k$ converges.
10. Find the radius of convergence of $\sum_{k=1}^{\infty} \frac{z^k}{k}$.
11. Define pole of order n . Give an example of a function with simple pole at $z = 1$.
12. Find the principal part in the Laurent series expansion about the origin of the function $f(z) = \frac{\sin z}{z^4}$.

Turn over

13. State Rouché's theorem.
14. Find the residue of $\frac{\sin z}{z}$ at $z = 0$.
15. How many zeroes are in the disc $|z| = 1$ for the function $f(z) = z^9 - 8z^2 + 5$.

(10 × 3 = 30 marks)

Section B*Answer at least five questions.**Each question carries 6 marks.**All questions can be attended.**Overall Ceiling 30.*

16. Check whether the function U is harmonic or not if so find its harmonic conjugate $U(x, y) = x^3 - 3xy^2 - 5y$.
17. Find all the solutions of the equation $\sin z = 5$.
18. State and prove Fundamental theorem of algebra.
19. State and prove Morera's theorem.
20. Find the Taylor's series expansion with centre $z_0 = 2i$ of $f(z) = \frac{1}{1-z}$.
21. Identify the singular points and classify them $f(z) = \frac{\sin z e^{\left(\frac{1}{z-1}\right)}}{z(1+z)}$.
22. Find residue of $e^{\left(\frac{1}{z}\right)}$ at $z = 0$.
23. Find $\int_{-\infty}^{\infty} \frac{1}{x^4 + 1} dx$.

(5 × 6 = 30 marks)

Section C (Essay Questions)*Answer any two questions.**Each question carries 10 marks.*

24. State and prove Cauchy Riemann Equation. Also state the sufficient condition for differentiability.
25. State and prove Cauchy's integral formula for derivatives.
26. Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for $1 < |z-2| < 2$.
27. State and prove Cauchy's residue theorem.

(2 × 10 = 20 marks)