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C 42788 (Pages : 2) Name.......................................

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2023

(CBCSS)

Mathematics

MTH 2C 07—REAL ANALYSIS—II

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Questions)

*Answer **all** questions.*
Each question carries 1 weightage.

1. Prove that Lebesgue outer measure is translation invariant.
2. Define cantor set and prove that it has a measure zero.
3. State a sufficient condition for a function f to be measureable and show that continuous functions on measureable domains are measureable.
4. Show that for a sequence of bounded measurable functions, uniform convergence is sufficient for passage of limit under integral sign.
5. Let f and g are integrable functions. State and prove Additivity over domains of integration.
6. Define Convergence in measure for a sequence of measurable functions and give the statement of Riesz Theorem.
7. Define Convex functions and state Chordal Slope Lemma.
8. Prove that every convergent sequence in a normed space is Cauchy.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **six** *questions, choosing* **two** *questions from each unit.*
Each question carries 2 weightage.

UNIT I

9. Prove that all countable sets are measureable.

10. Prove the existence of non measurable sets.

11. State and prove Simple Approximation Theorem.

UNIT II

12. State and prove countable Additivity of integration for integrable functions.

13. State and Prove Lebesgue dominated convergence Theorem.

14. State and Prove Vitali convergence Theorem for sequence of functions defined on any set E.

UNIT III

15. a) Define a cover of a set E in the sense of Vitali

    b) State Vitali Covering Lemma

    c) Define Upper and lower derivatives of a real valued function.

    d) State Lebesgue Theorem on Differentiability.

16. State and prove Jordan's Theorem for functions of bounded variation.

17. Prove that a convex function on (a, b) is differentiable except at countable number of points and its derivative is an increasing function.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *from the following four questions.*
Each question carries 5 weightage.

18. Define Lebesgue Outer Measure and prove that outer measure of an interval is its length.

19. Define measurability of functions and prove that if f and g are measurable function on E and are finite valued on E, then $f + g$ and fg are measurable.

20. State and prove Fatou's Lemma and Monotone Convergence Theorem.

21. State and prove Holder inequality and Minkowski's Inequality.

(2 × 5 = 10 weightage)