

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 06—ABSTRACT ALGEBRA

Time : Three Hours

Maximum : 120 Marks

Section A

*Answer all the twelve questions.**Each question carries 1 mark.*

1. Fill in the blanks : The total number of commutative binary operations on a set of n elements is _____.
2. Fill in the blanks : The number of elements in the ring $M_2(\mathbb{Z}_3)$ is _____.
3. Fill in the blanks : The least value of n such that a group G of order n is non-abelian is _____.
4. Define a group.
5. Give an example of a finite integral domain.
6. Define skew fields.
7. Calculate the order of the permutation $\mu = (1)(2)(3)$ in S_4 .
8. Solve : $-3x + 2 = 4$ in the group $\langle \mathbb{Z}_6, +_6 \rangle$.
9. Show that the identity element in a group is unique.
10. How many left cosets are there for $p\mathbb{Z}$ in \mathbb{Z} if p is a prime.
11. What is a Klein group ?
12. Give a group theoretic definition of greatest common divisor of two positive integers.

(12 × 1 = 12 marks)

Section B

*Answer any ten out of fourteen questions.**Each question carries 4 marks.*

13. If H is a finite non-empty subset of a group G , establish that H will be a subgroup if it is closed under the binary operation in G .
14. Show that a group is a finite group if it has finite number of subgroups.

Turn over

15. Show that every cyclic group is abelian.
16. Find all group homomorphism from \mathbb{Z} into itself.
17. Let G be a group of order pq where p and q are primes. Show that every proper subgroup of G is cyclic.
18. Let S be a set and let f, g and h be functions mapping S into S . Prove that $f^*(g^*h) = (f^*g)^*h$ where the binary operation $*$ is the function composition.
19. Is the union of two subgroups a subgroup? Justify your claim.
20. Show that the coset multiplication given by $(aH)(bH) = abH$ is a well defined operation when H is a normal subgroup of G .
21. Draw the subgroup diagram for \mathbb{Z}_{18} .
22. Show that any finite cyclic group of order n is isomorphic to \mathbb{Z}_n .
23. Find a group isomorphic to the Klein group other than the Klein group. Establish that it is so.
24. Give any necessary and sufficient condition for a ring R to have no zero divisors. Justify your claim.
25. Is \mathbb{Q} , the set of rationals, the field of quotients for integers? Give reasons to establish your assertion or denial.
26. Show that factor group of a cyclic group is always abelian.

(10 × 4 = 40 marks)

Section C

Answer any **six** out of nine questions.

Each question carries 7 marks.

27. Draw the group table for the dihedral group D_4 . Is D_4 a cyclic group? Justify your claim.
28. Define kernel of a group homomorphism and show that it is a normal subgroup of the domain of the homomorphism.
29. Define order of an element in any group G . Show that in a finite group G , order of any element divides order of G .
30. Show that every permutation σ of a finite set is a product of disjoint cycles.
31. Prove or disprove: Every finite integral domain is a field.
32. Define the alternate group A_n . Show that it is a normal subgroup and find the group isomorphic to S_n/A_n .

33. Let G be a finite group in which for each positive integer m , the number of solutions of $x^m = e$ is at most m . Then show that G is cyclic.
34. If $\langle R, + \rangle$ is an abelian group, show that $\langle R, +, \cdot \rangle$ is a ring if $a \cdot b$ is defined as 0 for all $a, b \in R$.
35. Prove in some detail that every field L containing an integral domain D contains the field of quotients of D .

(6 × 7 = 42 marks)

Section D

*Answer any two out of three questions.
Each question carries 13 marks.*

36. (a) If $\phi: G \rightarrow G'$ is a group homomorphism then show that $\phi[H] \leq G'$ whenever $H \leq G$.
- (b) Find the index of the subgroup generated by $\sigma = (1, 5, 3, 4)(2, 3)$ in S_5 .
37. (a) Prove that the converse of the Lagrange's theorem need not be true.
- (b) Express $\sigma = (1\ 2\ 3)(1\ 3\ 4)^2 \in S_4$ as a product of disjoint cycles.
38. (a) Define rings and ring homomorphisms. Show that the ring of real numbers and complex numbers are not isomorphic.
- (b) Find all the units in the ring $\langle \mathbb{Z}_{18}, +_{18}, \times_{18} \rangle$.

(2 × 13 = 26 marks)