

**THIRD SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION
NOVEMBER 2020**

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 80 Marks

Part A

*Answer all the twelve questions.
Each question carries 1 mark.*

1. Evaluate $\int_0^{\pi/6} \tan 2x \, dx$.
2. Define an alternating series.
3. Find the vertices of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.
4. Find the Taylor polynomial of order zero generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$.
5. Evaluate $\frac{d}{dt}(\tan h \sqrt{1+t^2})$.
6. Examine whether $\sum_{n=1}^{\infty} n^2$ converges or diverges.
7. Find the directive of the parabola $y^2 = 10x$.
8. Define absolute convergence.
9. Find y if $\ln y = 3t + 5$.
10. Find the parametric equation of the circle $x^2 + y^2 = a^2$.

Turn over

11. Examine whether $x^2 + xy + y^2 - 1 = 0$ represents a parabola, ellipse or hyperbola.

12. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$.

(12 × 1 = 12 marks)

Part B

Answer any **nine** questions.
Each question carries 2 marks.

13. For what values of x do the series $\sum_{n=0}^{\infty} n! x^n$ converges.

14. Evaluate $\frac{d}{dx} \ln_{10}(3x+1)$.

15. Evaluate $\int \frac{\log_2 x}{x} dx$.

16. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$.

17. Find $\frac{dy}{dx}$ if $y = x^x, x > 0$.

18. Graph the set of points whose polar co-ordinates satisfy the conditions $1 \leq r \leq 2$ and $0 \leq \theta \leq \frac{\pi}{2}$.

19. Examine whether the series :

$5 + \frac{2}{3} + 1 + \frac{1}{7} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \dots + \frac{1}{k!} + \dots$ converges.

20. Evaluate $\int_0^{\ln 2} 4e^x \sin hx \, dx$.

21. Show that $\ln x$ grows slower than x as $x \rightarrow \infty$.

22. Examine whether $\sum_{n=1}^{\infty} (-1)^{n+1}$ converges or diverges.

23. Evaluate $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x}$.

24. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta} d\theta$.

(9 × 2 = 18 marks)

Part C

Answer any **six** questions.
Each question carries 5 marks.

25. The standard parametrization of the circle of radius 1 centered at the point (0, 1) in the xy -plane is $x = \cos t$, $y = 1 + \sin t$, $0 \leq t \leq 2\pi$. Use the parametrization to find the area of the surface swept out by revolving the circle about the x -axis.

26. Find the centroid of the first quadrant of the astroid $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq 2\pi$.

27. Show that $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} = e$.

28. Using Integral test show that the p -series :

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \dots + \frac{1}{n^p} + \dots \text{converges if } p > 1 \text{ and diverges if } p \leq 1.$$

29. Find the Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.

30. Find the length of the Cardioid $r = 1 - \cos\theta$.

31. Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

Turn over

32. Graph the curve $r^2 = 4 \cos \theta$.

33. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n! n!}$.

(6 × 5 = 30 marks)

Part D

*Answer any two questions.
Each question carries 10 marks.*

34. Find the area of the region in the plane enclosed by the Cardioid $r = 2(1 + \cos \theta)$.

35. Show that the Maclaurin's series for $\cos x$ converges to $\cos x$ for every value of x .

36. Using Integral test examine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

(2 × 10 = 20 marks)