

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS-UG)

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 120 Marks

Part A

*Answer all the twelve questions.**Each question carries 1 mark.*

1. Find the domain and range of $f(x, y) = \sin xy$.
2. Evaluate $\lim_{(x, y) \rightarrow (0, 0)} \frac{x}{y}$.
3. Find the gradient of $\phi(x, y, z) = xyz$.
4. Compute the divergence of $\vec{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$.
5. In what direction the directional derivative of a function becomes maximum ?
6. What do you mean by an irrotational vector ?
7. What is the linearization of the function $f(x, y, z)$ at the point $(1, 2, 3)$?
8. Find the total differential of u if $u = \ln(x^2 + y^2 + z^2)$.
9. Fill in the blanks : If \vec{f} and \vec{g} are irrotational vector point functions, then $\nabla \cdot (\vec{f} \times \vec{g}) = \dots$
10. State the Normal form of Green's theorem in the plane.
11. Fill in the blanks : If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then $r^n\vec{r}$ is solenoidal if $n = \dots$
12. State Gauss's Divergence theorem mentioning all the assumptions involved in it explicitly.

(12 × 1 = 12 marks)

Part B

*Answer any ten questions.**Each question carries 4 marks.*

13. Find the vector normal to the surface $\phi(x, y, z) = x^3 + y^3 + z^3$ at $(1, -1, 1)$.
14. Evaluate $\lim_{(x, y) \rightarrow (0, 0)} \frac{y-x}{y+x}$.

Turn over

15. If $z^3 - xy + yz + y^3 - 2 = 0$ then, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1, 1, 1)$.
16. Find the total derivative of $u = x^3 + y^3$ with respect to t if $x = \cos t, y = \sin t$.
17. Compute the average value of the function $f(x, y, z) = xyz$ over the boundary of the cube $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$.
18. Linearize the function $f(x, y, z) = x^2 - xy + 3\sin z$ at $(2, 1, 0)$.
19. Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $(1, 2, 3)$ in the direction of $3\vec{i} + 4\vec{j} + 5\vec{k}$.
20. Find the circulation of $\vec{f} = (x, y)\vec{i} + x\vec{j}$ around the unit circle centered at the origin.
21. Find the value of λ which makes the following vector $\vec{f} = (\lambda xy - z^3)\vec{i} + ((\lambda - 2)x^2)\vec{j} + ((1 - \lambda)xz^2)\vec{k}$ is irrotational.
22. Verify whether the differential $e^x \cos y dx + (xz - e^x \sin y) dy + (xy + z) dz$ is exact or not.
23. If \vec{f} and \vec{g} are irrotational, then show that $\vec{f} \times \vec{g}$ is solenoidal.
24. Evaluate $\int_0^{2x} \int_{x^2}^{2x} (4x + 2) dy dx$ by changing the order of integration.
25. Prove that if a function $f(x, y)$ is differentiable at the origin, then it is continuous at the origin.
26. Evaluate the integral $\int_c xy dy - y^2 dx$ where c is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

(10 × 4 = 40 marks)

Part C

Answer any **six** questions.
Each question carries 7 marks.

27. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$.
28. Find $\nabla f(r)$, if $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.
29. Show that the work done by the force field $\vec{f} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is independent of the path joining the points $(-1, 3, 9)$ and $(1, 6, -4)$.

30. Test the continuity of $f(x, y)$ defined by $f(x, y) = \frac{x^2 + y^2}{x + y}$, $(x, y) \neq (0, 0)$ and $f(x, y) = 0$, $(x, y) = (0, 0)$.
31. Find the equation to the tangent plane and normal line to the surface $f = (x, y, z) = x^2 + y^2 + z^2 - 9 = 0$ at the point $(1, 2, 4)$.
32. Evaluate the area enclosed by the region $x^2 + y^2 = 4$, $y = 1$, $y = \sqrt{3x}$ in the (x, y) plane, using double integrals.
33. Find the Local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.
34. Evaluate the volume of the ellipsoide $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
35. Show that $\vec{f} = (x + 2y + 4z)\vec{i} + (2x - 3y - z)\vec{j} + (4x - y - 2z)\vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

Answer any **two** questions.
Each question carries 13 marks.

36. (a) Verify Gauss's divergence theorem for $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere of radius a centered at the origin.
(b) State the Fundamental theorem of line integration.
37. (a) Evaluate the line integral $\int_C \vec{f} \cdot d\vec{r}$ where C is the boundary of the triangle with vertices $(0,0,0)$, $(1,0,0)$ $(1,1,0)$.
(b) Find (Curl Curl) \vec{f} , if $\vec{f} = x^2y\vec{i} - 2xz\vec{j} + 2yz\vec{k}$.
38. (a) If S is a closed surface enclosing a volume V then prove that $\iint_S \text{curl } \vec{f} \cdot \hat{n} dS = 0$.
(b) Verify Green's theorem for $\int_C (xy + y^2) dx + x^2 dy$, where C is the curve joining $y = x^2$ and $y = x$.

(2 × 13 = 26 marks)