

D 140194

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Name.....

Reg. No.....

**SIXTH SEMESTER (CBCSS—U.G.) DEGREE EXAMINATION
APRIL 2026**

Mathematics

MTS6B12—CALCULUS OF MULTI VARIABLE

(2020 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Ceiling is 25.*

1. What is a level curve of a function of two variables ?
2. Let $f(x, y) = x^2 + 3xy - 2x + 3$. Find $f(2, 1)$ and $f(x, y + k)$.
3. Evaluate $\lim_{(x, y, z) \rightarrow \left(\frac{\pi}{2}, 0, 1\right)} \frac{e^{2y} (\sin x + \sin y)}{1 + y^2 + z^2}$.
4. Show that the function $u(x, y) = e^x \cos y$ is harmonic in the xy plane.
5. Find the directional derivative of $f(x, y) = 4 - 2x^2 - y^2$ at the point $(1, 1)$ in the direction of the unit vector u that makes an angle of $\frac{\pi}{3}$ radians with the positive x -axis.
6. Find the critical points of $f(x, y) = 3 - \sqrt{x^2 + y^2}$.
7. State Lagrange's theorem.
8. Find an approximation for $\iint_R (x - 4y) dA$, where $R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1\}$, using Riemann sum of $f(x, y) = x - 4y$ over R with $m = n = 2$ and taking evaluation point (x_{ij}^*, y_{ij}^*) to be the center of R_{ij} .

Turn over

9. Evaluate $\int_0^1 \int_0^2 3x^2 y \, dy dx$.
10. State Fubini's theorem for rectangular regions.
11. Find the moments of inertia with respect to the x -axis, the y axis and the origin of a homogeneous disk of mass m and radius a , centered at the origin.
12. Define divergence of a vector field. Find the divergence of $F(x, y) = xi + yj$.
13. Find curl of $F(x, y, z) = xi + yj$.
14. Let $F = Pi + Qj + Rk$ be a vector field in space, and suppose that P , Q and R have continuous second order partial derivatives. Show that $\operatorname{div} \operatorname{curl} F = 0$.
15. State Green's theorem.

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 35.

16. A storage tank has the shape of a right circular cylinder. Suppose that the radius and height of the tank are measured at 1.5 ft and 5 ft, respectively, with a possible error of 0.05 ft and 0.1 ft, respectively. Use differentials to estimate the maximum error in calculating the capacity of the tank.
17. Let $W = x^2y - xy^3$, where $x = \cos t$ and $y = e^t$. Find $\frac{dw}{dt}$ and its value at $t = 0$.
18. Find the relative extrema of $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.
19. Find the volume of the solid S lying under the graph of the surface $z = x^3 + 4y$ and above the region R in the xy plane bounded by the line $y = 2x$ and the parabola $y = x^2$.
20. Find the surface area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$.

21. Let f be a scalar function, and let $\mathbf{F}$ be a vector field. If f and the components of $\mathbf{F}$ have first order partial derivatives, show that $\operatorname{div} (f \mathbf{F}) = f \operatorname{div} \mathbf{F} + \mathbf{F} \cdot \nabla f$.
22. Find the work done by the force field $\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ in moving a particle along the helix C described by the parametric equations $x = \cos t$, $y = \sin t$ and $z = t$ from $(1, 0, 0)$ to $\left(0, 1, \frac{\pi}{2}\right)$.
23. Evaluate $\oint_C x^2 dx + (xy + y^2) dy$, where C is the boundary of the region R bounded by the graphs of $y = x$ and $y = x^2$ and is oriented in a positive direction.

Section C

Answer any two questions.

Each question carries 10 marks.

Maximum 20 marks.

24. Let $w = f(x, y)$, where f has continuous second order partial derivatives and let $x = r^2 + s^2$ and $y = 2rs$. Find $\frac{\partial^2 w}{\partial r^2}$.
25. Find the absolute extreme values of $f(x, y) = 2x^2 + y^2 - 2y + 1$ subject to the constraint $x^2 + y^2 \leq 4$.
26. Evaluate $\iiint_T \sqrt{x^2 + z^2} dv$, where T is the region bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y + z = 2$ and $y = 0$.
27. State Stoke's theorem. Verify Stoke's theorem for the case in which $\mathbf{F}(x, y, z) = 3z\mathbf{i} + 2x\mathbf{j} + y^2\mathbf{k}$, S is the part of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$, and C is the trace of S on the xy plane.

(2 × 10 = 20 marks)