C 61250 (Pages : 3) Name.......................................

Reg. No.......................................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCBCSS–UG)

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. State the Existence and Uniqueness theorem for initial value problem.
2. Define and give an example of an even function.
3. What do you mean by a non-linear differential equation ?
4. Solve $y'' - y' - 2y = 0$.
5. Define a unit step function.
6. State the existence theorem for Laplace transforms.
7. Find $L^{-1}\left(\frac{a}{s^2 - a^2}\right)$.
8. Find $L\left(t^{-1/2}\right)$.
9. Define a rectangular wave.
10. Write the 2-dimensional Poisson equation.
11. Give a formula for an error for Simpson's rule.
12. Write the formula for Runge Kutta method.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find the particular integral for $y' + 4y = 8x^2$.
14. Find a basis for the solution of the differential equation $y'' + y = 0$.

Turn over

15. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at}F(t)$.

16. Solve $3y'' - 8y' - 3y = 0, y(-3) = 1, y(3) = \left(\frac{1}{e^2}\right)$.

17. Find $L\left(e^{-\alpha t} \cos \beta t\right)$.

18. If $f(x)$ is a periodic function of x of period p, show that $f(ax), a \neq 0$, is a periodic function of x of period $\frac{p}{a}$.

19. Find the Fourier series of $f(x) = x + |x|, -\pi < x < \pi$.

20. Show that $u = e^{-t} \sin x$ is a solution of heat equation.

21. Apply Picards iteration to solve $y' = y - x^2, y(0) = 1$ also find y (0.1) and y (0.2).

22. Evaluate $\int_{-3}^{3} x^4 dx$ using Simpson's rule.

23. What do you mean by convolution ?

24. Evaluate $\int_{0}^{6} \frac{1}{1+x} dx$ by Trapezoidal rule.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Solve $\left(4x^2D^2 + 12xD + 3\right)y = 0$.

26. Find a general solution of the differential equation $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$.

27. Find the Laplace transform of $(t-1)^2 u(t-1)$.

28. Find $L^{-1}\left(\frac{4\left(e^{-2s}-2^{e-5s}\right)}{s}\right)$.

29. Solve $u_{xy} = u_x$.

30. Find the Fourier series of $f(x)=\begin{cases}-k, \text{ if } -\pi < x < 0; \\ k, \text{ if } 0 < x < \pi,\end{cases}$ and $f(x+2\pi)=f(x)$.

31. Given $y' = -y, y(0) = 1$. Find the value of y at $x = (0.01)(0.01)(0.04)$ by improved Euler method.

32. Find approximate solution to $y' + y = e^x, y(0) = 0$.

33. Evaluate $\int_4^{5.2} \log_e x \, dx$ using Simpson's rule.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve $x^2y'' - 2xy' + 2y = x^3 \sin x$.

35. Solve the integral equation $y(t) = t + \int_0^t y(\tau)\sin(t-\tau)\,d\tau$.

36. Find the Fourier series $f(x)=\begin{cases}x+x^2, \text{ if } -\pi < x < \pi; \\ \pi^2, \quad \text{if } x = \pm\pi.\end{cases}$

(2 × 10 = 20 marks)