C 30802 (Pages : 2) Name.....

Reg. No.....

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION, NOVEMBER 2017

(UG-CCSS)

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each question carries ¼ weightage.

1. What is the order of $y''' + 7.y''^2 + y' = \cos t$?
2. Is $y' = x^3\left(y - y^2\right)$ separable ?
3. Check for exactness : $3x^2 - 2xy + 2 + y'\left(6y^2 - x^2 + 3\right) = 0$.
4. Solve : $y'' - 9y = 0$.
5. State Abd's theorem.
6. Find the Wronskian of $y_1 = \cos t, y_2 = \sin t$.
7. Find $\mathrm{L}\left\{e^{-7t}\right\}$.
8. State True *or* False. The Laplace transform is a linear operator.
9. What is L {sin $4t$} ?
10. What is the fundamental period of cos $5t$?
11. What is the heat conduction equation ?
12. Is the following function even, odd or neither : $x^3 - 4x$.

(12 × ¼ = 3 weightage)

Section B

Answer **all** *questions.*
Each question carries 1 *weightage.*

13. Verify that $y = 3t + t^2$ is a solution of $ty' - y = t^2$.
14. State the existence and uniqueness theorem for first order initial value problems.
15. Verify whether y^2 is an integrating factor of $\frac{dx}{y} + \frac{2x}{y^2}dy = 0$.
16. Solve $y'' + 4y' - 12y = e^{3t}$
17. Solve $y''' - y = 0$.

Turn over

18. Find L {sinh at}.

19. Show that convolution is commutative.

20. Show that the sum of two odd functions is odd.

21. Graph the sawtooth wave function.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*
Each question carries 2 weightage.

22. Find an integrating factor and solve : $y' = e^{2x} + y - 1$.

23. Solve the initial value problem : $(3x+8)\left(y^2+4\right)dx - 4y\left(x^2+5x+6\right)dy = 0,\ y(1) = 2$.

24. Solve the initial value problem : $y'' - 2y' - 3y = 3te^{2t},\ y(0) = 1,\ y'(0) = 0$.

25. Solve $y'' + 4y = \sin^2 t$.

26. Find $L^{-1}\left\{\dfrac{3s}{s^2 - s - 6}\right\}$.

27. Find L {t sin at}.

28. Find the Fourier cosine series of :

$$f(x) = \begin{cases} 1, 0 < x < 1 \\ 0, 1 < x < 2 \end{cases}, f \text{ of period } 4.$$

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*
Each question carries 4 weightage.

29. Solve by the method of variation of parameters $y'' + 4y = 3 \operatorname{cosec} 2t,\ 0 < t < \pi/2$.

30. (a) Using convolution, evaluate $L^{-1}\left\{\dfrac{1}{s\,(s+2)^3}\right\}$.

(b) Use Laplace transforms to solve $y'' - 2y' + y = 0,\ y(0) = 0,\ y'(0) = 1$.

31. Find the Fourier series expansion of

$$f(x) = \begin{cases} O, -L < x < O \\ L, O < x < L \end{cases}, f \text{ of period } 2L.$$

Hence deduce that $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \ldots$.

(2 × 4 = 8 weightage)