

D 143209

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Name.....

Reg. No.....

**FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY)
EXAMINATION, APRIL 2026**

(CBCSS)

Mathematics

MTH4E11—GRAPH THEORY

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A*Answer all the questions.**Each question carries a weightage 1.*

1. Distinguish between a trail and path. Illustrate with an example.
2. Define block and bridge of a simple graph. Illustrate with an example.
3. Define a tree. Prove that a (p, q) graph G is a tree if and only if $p = q + 1$.
4. Define diameter of a connected graph. Illustrate with an example.
5. Define chromatic number $\chi(G)$ of a graph G . Prove that $\chi(K_n) = n$.
6. Prove that in a bipartite graph G with $\delta > 0$, the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering.
7. Define line independence number $\beta_1(G)$ of a graph G . Prove that $\beta_1(K_p) = \left\lceil \frac{p}{2} \right\rceil$.
8. Prove that in a k -critical graph ($k \geq 2$), no vertex cut is a clique.

(8 × 1 = 8 weightage)

Turn over

Part B

*Answer any **two** questions from each module.*

Each question carries a weightage 2.

MODULE I

9. Prove that in a tree any *two* vertices are connected by a path.
10. Prove that a graph G is a cut edge of G if and only if e contained no cycle of G .
11. Prove that the number of spanning trees of a complete graph K_n is n^{n-2} .

MODULE II

12. Show that a matching M in G is a maximum matching if and only if G contains no M -augmenting path.
13. Prove that for any graph G of order n , $\alpha + \beta = n$, where α is the independence number and β is the covering number of G .
14. Prove that every 3-regular graph without cut edges has a perfect matching.

MODULE III

15. If the graph G is k -critical, then prove that $\delta \geq k - 1$.
16. Prove that K_5 is non-planar.
17. State and prove Euler formula for a connected plane graph G .

($6 \times 2 = 12$ weightage)

Part C

*Answer any **two** questions.*

Each question carries a weight of 5.

18. Prove that a non-empty connected graph is Eulerian if and only if it has no vertex of odd degree.

19. Explain the Chinese postman problem.
20. Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge coloring in which both colors are represented at each vertex of degree at least two.
21. Prove that every planar graph is 5-vertex colourable.

($2 \times 5 = 10$ weightage)