D 131478 **(Pages : 3)** **Name.....**

Reg. No.....

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2025

Mathematics

MTS 3C 03—MATHEMATICS—3

(2020—2023 Admissions)

Time : Two Hours Maximum : 60 Marks

Part A

All questions can be attended.
Each question carries 2 marks.
Overall Ceiling is 20.

1. Graph the curve C that is traced by a point P whose position is given by

   $r(t) = \cos 2t\, i + \sin t\, j$, where $0 \le t \le 2\pi$.

2. If $r(t) = \left(t^3 - 2t^2\right) i + 4t\, j + e^{-t}\, k$ then find $r''(t)$.

3. Describe the level surfaces of the function $f(x, y) = x + 2y$.

4. If $\mathrm{F} = \left(x^2y^3 - z^4\right) i + 4x^5y^2z\, j + y^4z^6k$, find div F.

5. Evaluate $\int xy^2\, dx$ on the quarter-circle C defined by $x = 4\cos t$, $y = 4\sin t$, $0 \le t \le \frac{\pi}{2}$.

6. Find $\int_C ydx + xdy$ on the curves $y = x^4$ between $(0, 0)$ and between $(1, 1)$.

7. Convert $\left(-\sqrt{2}, \sqrt{2}, 1\right)$ in rectangular coordinates to cylindrical co-ordinates.

8. Find the values of $\ln i$.

Turn over

9. Prove that $\cosh z = \cosh x \cos y + i \sinh x \sin y$.

10. Evaluate $\oint \frac{1}{z} dz$, where C is the circle $x = \cos t, y = \sin t, 0 \le t \le 2\pi$.

11. Evaluate $\oint_C \frac{dz}{z^2}$, where C is the ellipse $(x-2)^2 + \frac{(y-5)^2}{4} = 1$.

12. State Liouvilles Theorem and Fundamental Theorem of Algebra.

Part B

All questions can be attended.
Each question carries 5 *marks.*
Overall Ceiling is 30.

13. Find the directional derivative of $F(x, y, z) = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction of $6i + 2j + 3k$.

14. Find parametric equations for the normal line to the surface of $x^2 - 4y^2 + z^2 = 16$ at $(1, -1, 5)$.

15. A lamina has the shape of the region in the first quadrant that is bounded by the graphs of $y = \sin x, y = \cos x$, between $x = 0$ and $4x = \frac{\pi}{4}$. Find its center of mass if the density is $\rho(x, y) = y$.

16. Evaluate $\oint_C (x^2 - y^2) dx + (2y - x) dy$, where C consists of the boundary of the region in the first quadrant that is bounded by the graphs of $y = x^2$ and $y = x^3$.

17. Find the volume of the solid in the first octant bounded by the graphs of $z = 1 = y2, y = 2x$ and $x = 3$.

18. Compute z^3 for $z = 1 - \sqrt{3}i$.

19. Evaluate $\oint_C \frac{5z + 7}{z^2 + 2z - 3} dz$, where C is the circle $|z - 2| = 2$.

Part C

Answer any **one** *question.*
The question carries 10 *marks.*

20. Verify Stokes theorem. Assume that the surface S is oriented upward. Given $F = 5y\,i - 5x\,j + 3\,k$; S that portion of the plane z = 1 within the cylinder $x^2 + y^2 = 4$.

21. Find the volume of the solid in the first octant bounded by the graphs of $z = 1 - y^2$, $y = 2x$, and $x = 3$.

(1 × 10 = 10 marks)