

D 40043

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Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS-UG)

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the twelve questions.

Each question carries 1 mark.

1. Find an interval of unit length which contains the smallest positive root of the equation $x^3 - 2x - 5 = 0$.
2. State Newton's forward interpolation formula.
3. Evaluate $\Delta^2 ab^x$, interval of differencing being unity.
4. Write the relation between divided differences and forward differences.
5. Form the table of backward differences of the function $f(x) = x^3 + 5x - 7$ for $x = -1, 0, 1, 2, 3, 4, 5$.
6. Give the names of two interpolation formula applicable for unequally spaced values of the argument.
7. What do you mean by inverse interpolation ?
8. Given a set of n -values of (x, y) , what is the formula for computing $\left[\frac{dy}{dx}\right]_{x_0}$?
9. Give Simpson's 1/3 -rule of integration.
10. What is complete pivoting ?
11. In numerical integration, what should be the number of intervals to apply Simpson's 1/3- rule and by Simpson's 3/8-rule.
12. Write Milne's corrector formula.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any ten out of fourteen questions.
Each question carries 4 marks.

13. Find the iterative method based on the Newton-Raphson method for finding \sqrt{N} where N is a positive real number. Apply the method to $N = 18$ to obtain the results correct to two decimal places.
14. Prove that (i) $\nabla = I - E^{-1}$; (ii) $\delta = \Delta E^{1/2}$, where E is the shift operator and δ is the central differential operator.
15. Certain corresponding values of x and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843) and (307, 2.4871). Find $\log_{10} 301$.
16. Find the missing values in the following table :

x	45	50	55	60	65
y	3.0	-	2.0	-	-2.4

17. Construct the divided difference table for the following data :

x	-1	0	3
$f(x)$	-4	-5	16

Determine the approximate value of $f(1)$ using divided difference interpolation.

18. Find a polynomial satisfied by $(-4, 1245)$, $(-1, 33)$, $(0, 5)$, $(2, 9)$ and $(5, 1335)$.
19. Obtain the first derivative of \sqrt{x} at $x = 15$ from the table :

x	15	17	19	21	23	25
\sqrt{x}	3.873	4.123	4.359	4.583	4.796	5

20. Find, from the following table, the area bounded by the curve and the x -axis from $x = 7.47$ to $x = 7.52$:

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

21. Evaluate the integral $\int_0^{\pi/2} \sin x \, dx$, using Simpson's (3/8)-rule.
22. Solve the system of equations $3x + y - z = 3$; $2x - 8y + z = -5$; $x - 2y + 9z = 8$, using Gauss-elimination method.
23. Decompose the matrix $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ in the form LU.
24. Solve by Jacobi's iteration method, the equations $20x + y - 7z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$.
25. Find by Taylor's series method the value of y at $x = 0.1$ correct to five places of decimals from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$.
26. Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$ evaluate $y(1.4)$ by Milne's predictor-corrector method.

(10 × 4 = 40 marks)

Section C

Answer any six out of nine questions.
Each question carries 7 marks.

27. Using Ramanujan's method find a real root of the equation :

$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \dots = 0.$$

28. Find the 7th term and the general term of the series 3, 9, 20, 38, 65, _____.
29. The equation $f(x) = \log_e x - x + 3 = 0$ has a root in the interval (4, 5). Obtain the root correct to three decimal places using regula-falsi method.

Turn over

30. The following table of the function $f(x) = e^{-x}$ is given :

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

- (i) Using Gauss forward central difference formula compute $f(0.55)$.
 (ii) Using Gauss backward central difference formula compute $f(0.45)$.
31. From the following table, find x , correct to two decimal places, for which y is maximum and find this value of y .

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

32. Derive Simpson's 1/3- rule using the method of undetermined coefficients.
33. Tabulate $y = x^3$ for $x = 2, 3, 4$ and 5 , and calculate the cube root of 10 correct to three decimal places.

34. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ using Gaussian elimination and use the result to

solve the system of equations : $3x + 2y + 4z = 7, 2x + y + z = 7, x + 3y + 5z = 2$.

35. Using Euler's method, find an approximate value of y corresponding to $x = 0.1$, given that $\frac{dy}{dx} = x^2 + y, y(0) = 1$. (6 × 7 = 42 marks)

Section D

*Answer any two out of three questions.
Each question carries 13 marks.*

36. (a) A rod is rotating in a plane about one of its ends. If the following table gives the angle θ radians which the rod has turned for different values of time t seconds, find its angular velocity and angular acceleration when $t = 0.7$ seconds.

t seconds	0.0	0.2	0.4	0.6	0.8	1.0
θ radians	0.0	0.12	0.48	1.10	2.0	3.20

- (b) Find the value of $\int_1^2 \frac{dx}{x}$ by Trapezoidal rule. Hence find approximate value of $\log_e 2$.

37. Solve the system of equations $x - y + 5z = 5; 2x - 3y + z = 0; x + 3y + 7z = 11$ by LU decomposition.

38. (a) Solve the following by Euler's modified method : $\frac{dy}{dx} = \log_{10}(x + y), y(0) = 2$ at $x = 1.2$ and 1.4 with $h = 0.2$.

- (b) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$. (2 × 13 = 26 marks)