

D 131477

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Name.....

Reg. No.....

**THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2025**

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(2020–2023 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

**Section A**

*All questions can be attended.  
Each question carries 2 marks.  
Ceiling is 25.*

1. Differentiate the function  $f(x) = \log\left(\frac{\log(x)}{x^2}\right)$ .
2. Find the derivative of  $y = \log(|\operatorname{cosec}(x) - \cot(x)|)$ .
3. Find  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$ .
4. Show that  $\cosh^{-1} x = \log(x + \sqrt{x^2 - 1})$ .
5. Find  $\lim_{n \rightarrow \infty} \frac{n+1}{\sqrt{n^2 + n + 1}}$ .
6. Determine whether the sequence  $a_n = \frac{\sin(1/n)}{n}$  converges or diverges. If it converges, find the limit.
7. Express  $0.\overline{784784} \dots$  as a rational number.

**Turn over**

8. Show that the series  $\sum_1^{\infty} (3)^{2n} 5^{1-n}$  is divergent.
9. By using the power series expansion of  $e^{-x}$ , show that  $\frac{d}{dx}(e^{-x}) = -e^{-x}$ .
10. Find the Taylor series expansion of  $\sin x$  about  $x = \pi/2$ .
11. Find the parametric equation of the curve whose rectangular equation is  $xy = 1$ .
12. Show that the graph of the function  $f(x) = (x-4)^{2/3}$  has a vertical tangent at  $x = 4$ .
13. Find the distance between the point  $(1, -4, -3)$  and the plane  $2x - 3y + 6z + 1 = 0$ .
14. Find  $r(t)$  given that  $r'(t) = 3i + 2tj$  and  $r(1) = 2i + 5j$ .
15. Find the co-ordinates of point where the line  $r(t) = (2+t)i + (1-2t)j + 3tk$  intersects the  $xz$  plane.

(Ceiling 25)

**Section B**

*All questions can be attended.  
Each question carries 5 marks.  
Overall Ceiling is 35*

16. Evaluate :

(i)  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 2x}$ .

(ii)  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin x}$ .

17. Use Logarithmic differentiation to find the derivative of  $y = \sqrt{x^x}$ .
18. Find  $\int \sin(x)e^x dx$ .
19. Show that the sequence  $a_n = \frac{n!}{n^n}$  converges and find its limit.

20. Find the interval of convergence and radius of convergence of the power series  $\sum_0^{\infty} n!x^n$ .
21. Find the Maclaurian series for  $\sinh x$  and determine its interval of convergence.
22. Find the arc length of the cardioid  $r = 1 + \cos \theta$ .
23. Find the curvature and radius of curvature of the curve  $r(t) = 2\sin(2t)i + \sin(t)j$  at  $t = \pi/2$ .

(Ceiling 35)

**Section C**

*Answer any two questions.  
Each question carries 10 marks.*

24. (i) Evaluate  $\int x^2 \log(x) dx$ .

(ii) Find  $\int \frac{1}{\sqrt{1+e^{-2x}}} dx$ .

(iii) Find  $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$ .

25. (i) Let C be the circle  $r(t) = \cos t + \sin t + 2k$ . Find T(t) and N(t) at  $t = \pi/4$ .

(ii) Identify and sketch the graph of the surface  $z = \frac{y^2}{4} - \frac{x^2}{9}$ .

26. (i) Find  $\frac{d^y}{dx^2}$  for  $x = \sin(t), y = \sin(2t)$ .

(ii) Find the area of the region in the first quadrant that is within the cardioid  $r = 1 - \cos \theta$ .

27. (i) Find the equation of the paraboloid  $z = x^2 + y^2$  in cylindrical and spherical co-ordinates.

(ii) Find the spherical co-ordinates of the point that has rectangular co-ordinates  $(4, -4, 4\sqrt{6})$ .

(2 × 10 = 20 marks)