

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2019

(CUCSS—PG)

Mathematics

MT 4E 11—GRAPH THEORY

(2016 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A (Short Answer Type)*Answer all questions.**Each question has weightage 1.*

1. With usual notations, prove that $\epsilon = v - 1$ in a tree.
2. If T is a spanning tree of a connected graph G and e is an edge of G not in T , prove that $T + e$ contains a unique cycle.
3. Draw all spanning trees of K_4 .
4. Find a simple graph with $\delta = v - 3$ and $\kappa < \delta$.
5. Let G be a block with $v(G) \geq 3$. Prove that any two edges of G lie on a common cycle.
6. If a graph is 2-connected, prove that any two vertices of G are connected by at least two edge disjoint paths.
7. Let $S \subseteq V$. If S is an independent set of G , prove that $V \setminus S$ is a covering of G .
8. Define a Hamiltonian graph. Give an example.
9. Let G be a simple graph which is Hamiltonian. Let u and v be non-adjacent vertices in G such that $d(u) + d(v) \geq v$. Prove that $G + uv$ is Hamiltonian.
10. In a critical graph, prove that no vertex cut is a clique.
11. Verify whether the 3-cube has a perfect matching.
12. If G is bipartite, prove that $\chi' = \Delta$.

Turn over

13. Prove that every tournament has a directed Hamiltonian path.
 14. If G is a simple planar graph with $v \geq 3$, prove that $e \leq 3v - 6$.

(14 × 1 = 14 weightage)

Part B (Paragraph Type)*Answer any seven questions from the following ten questions.**Each question has weightage 2.*

15. Let G be a loopless simple graph with exactly one spanning tree T . Prove that $G = T$.
 16. Find a simple graph with $\delta = \frac{v}{2} - 1$ and $\kappa' < \delta$.
 17. Let T be a spanning tree of a connected graph G and e be any edge of T . Show that $\bar{T} + e$ contains a unique bond of G .
 18. Find the number of spanning trees in $K_{3,3}$, using a recursion formula.
 19. Define closure of a graph G . Show that $C(G)$ is well defined.
 20. Explain the Fleury's algorithm to construct a trail in a graph.
 21. If G is a k -regular bipartite graph with $k > 0$ show that G has a perfect matching.
 22. With usual notations, prove that $r(k, l) \leq \binom{k+l-2}{k-1}$.
 23. Find all the 2-critical and 3-critical graphs.
 24. Show that $K_{3,3}$ is non-planar.

(7 × 2 = 14 weightage)

Part C (Essay Type)*Answer any two questions from the following four questions.**Each question has weightage 4.*

25. With usual notations, prove that $\kappa \leq \kappa' \leq \delta$.
 26. If G is simple, prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.
 27. Show that, in any graph G , $\pi_k(G)$ is a polynomial in k of degree v with integer co-efficients, leading term k^v and constant term zero and the co-efficients are alternate in sign.
 28. Prove that a graph is planar if and only if it contains no subdivision of K_5 or $K_{3,3}$.

(2 × 4 = 8 weightage)