C 41700 (Pages : 15) Name.....

Reg. No.....

P.G./INTEGRATED P.G. ENTRANCE EXAMINATION, APRIL 2023

STATISTICS

Time : Two Hours Maximum : 300 Marks

Each question carries 4 marks.

1 mark will be deducted for each wrong answer.

1. The inverse of a square matrix exists if and only if it is.

   (a) Singular. (b) Non-singular.

   (c) Symmetric. (d) Skew-symmetric.

2. Let a non-singular square matrix A be such that $A^T = A^{-1}$, then A is :

   (a) A skew symmetric matrix. (b) Hermitan matrix.

   (c) Symmetric matrix. (d) An orthogonal matrix.

3. Rank of a matrix is :

   (a) The number of independent rows of the matrix.

   (b) The number of dependent rows of the matrix.

   (c) The sum of the diagonal elements.

   (d) Min {No. of rows, No. of columns}.

4. Let $g : A \to B$ be a function, then g^{-1}exists if and only if g is :

   (a) Injective. (b) Surjective.

   (c) Bijective. (d) Trijective.

5. Which of the following statement(s) is/are true ?

   I. If $f(x) = p(x) \cdot q(x)$, then $\lim_{x \to a} f(x) = f(a)$.

   II. If $f(x) = \frac{p(x)}{q(x)}$, then $\lim_{x \to a} f(x) = f(a)$.

   (a) I. (b) II.

   (c) Both I and II. (d) Neither I nor II.

6. A function is continuous at $x = c$ if and only if :

(a) $f(c)$ is defined.

(b) $\lim_{x \to c} f(x)$ exists.

(c) $\lim_{x \to c} f(x) = f(c)$.

(d) All the above.

7. Let $f(x) = \frac{x^3 + 2x^2 - 1}{x + 5}$, then the first derivative of $f(x)$ at $x = 0$ is :

(a) $\frac{1}{25}$.

(b) $-\frac{1}{25}$.

(c) $\frac{4}{25}$.

(d) $-\frac{4}{25}$.

8. The Maclaurin series expansion of e^{-x^2} is :

(a) $1 - x^2 + \frac{x^4}{2} - \frac{x^6}{3} + \frac{x^8}{4} + \ldots$

(b) $1 + x^2 + \frac{x^4}{2} + \frac{x^6}{3} + \frac{x^8}{4} + \ldots$

(c) $1 + x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} + \ldots$

(d) $1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \ldots$

9. $1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots + \frac{1}{4^k} + \ldots$ equals :

(a) $\frac{4}{3}$.

(b) $\frac{1}{3}$.

(c) $\frac{2}{3}$.

(d) $\frac{5}{3}$.

10. Consider the given series and find its correct properties from the given statements

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \ldots$$

(i) The given series is divergent.

(iii) The given series is convergent series.

(ii) This is a harmonic series

(iv) This series is a geometric series.

(a) The statements (i) and (ii) are true.

(b) The statements (ii) and (iii) are true

(c) The statements (iii) and (iv) are true.

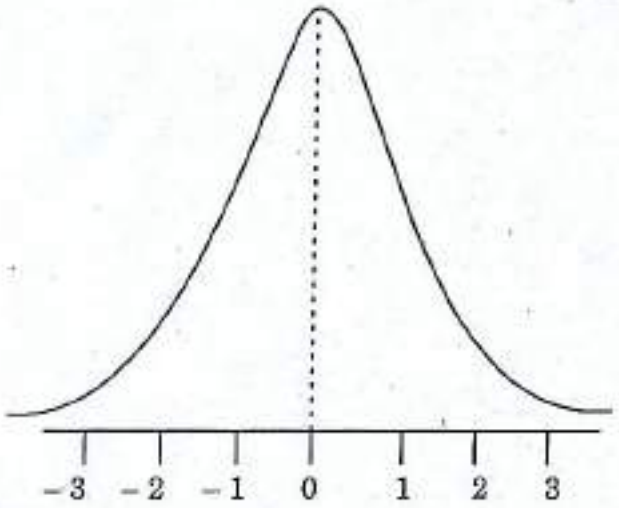
(d) The statements (i) and (iv) are true.

11. Find the value of $\int_1^4 (x+2)^2\,dx$.

(a) 36. (b) 63.

(c) 34. (d) 64.

12. Consider the statements based on the diagram and find the correct option.



(i) mean=median=mode.

(ii) mean>mode>median.

(iii) The given distribution is positively skewed.

(iv) The given distribution is symmetric.

(a) The statements (i) and (iii) are true.

(b) The statements (i) and (iv) are true.

(c) The statements (ii) and (iii) are true.

(d) The statements (ii) and (iv) are true.

13. The arithmetic mean of first 100 natural numbers is :

(a) 50.5. (b) 50.05.

(c) 50. (d) None of these.

14. Consider the data : 1, 4, 7, 12, 15, 20, 30, 50, ... 100. Let its arithmetic mean be "u". Then sum : $(1-u)+(4-u)+(7-u)+(12-u)+((15-u)+(20-u)+(30-u)+(50-u)+\ldots+(100-u)$ is :

(a) 1. (b) -1.

(c) 100. (d) 0.

15. Find which of the given statement is true :

Statement 1 : Arithmetic mean is not affected by extreme values.

Statement 2 : Some distributions have multiple number of modes.

(a) Statement 1 is right and statement 2 is wrong.

(b) Statement 1 is wrong and statement 2 is right.

(c) Both the statements are right.

(d) None of the statements are true.

16. The ratio of standard deviation of a data to its arithmetic mean is :

(a) Co-efficient of variation.

(b) $\frac{\text{Co-efficient of variation}}{100}$.

(c) Co-efficient of variation × 100.

(d) Square root of co-efficient of variation.

17. Consider two sets of data :

X :	5	4	3	5	4	2
Y :	4	5	9	12	15	1

If the coefficient variation of X is greater than coefficient of variation of Y then,

(a) X is more consistent / homogenous than Y.

(b) Y is more consistent / homogenous than X.

(c) Both are equally consistent.

(d) None of the above.

18. The 5^{th} decile of a data is also known as :

(a) Mean. (b) Mode.

(c) Median. (d) Harmonic mean.

19. Find the wrong statement regarding the properties of Variance of a data is :

(a) Sum of squares of deviations of x from mean divided by n.

(b) Square of standard deviation

(c) Variance is independent on change of origin.

(d) Variance is independent on change of scale.

20. A distribution is said to be leptokurtic if :

(a) $\beta_2 < 3$. (b) $\beta_2 > 3$.

(c) $\beta_2 = 3$. (d) $\beta_2 \neq 3$.

21. Consider a random variable X with Var (X) = E (X^2) then:

Statement 1 : Mean = 0

Statement 2 : $E\left|X^2 + 5\right| = 0$.

(a) Statement 1 is right and statement 2 is wrong.

(b) Statement 1 is wrong and statement 2 is right.

(c) Both the statements are right.

(d) None of the statements are true.

22. Consider a set of data $x_1, x_2, ..., x_n$. If $\beta_1 = \beta_2$ and let $E(X - E(X))^4 = \mu_4$, $E(X - E(X))^3 = \mu_3$, $\mu_4 = \mu_3^2$. Then Variance of the data is :

(a) 2. (b) $\frac{1}{4}$.

(c) $\frac{1}{5}$. (c) 1.

23. Consider the following data :

X	:	45	6	2	7	50	20
Y	:	2025	46	4	49	2502	410

Here the Karl-Pearson coefficient of correlation between X and Y will be :

(a) Near to 1. (b) Near to – 1.

(c) Near to zero. (d) Will be more than 0.6.

24. If X and V are independent, the value of regression coefficient β_{XY} is :

(a) 1.

(b) -1.

(c) 0.

(d) Undefined.

25. If the regression coefficients $\beta_{XY} = -\beta_{XY}$ then :

(a) The two regression lines are perpendicular to each other.

(b) The two lines of regression are coincident.

(c) Both (a) and (b) are true.

(d) None of (a) and (b) are true.

26. Let "α" be correlation between two variables X_1 and X_2 out of a set of variables (X_1, X_2, X_3) by eliminating the common association of remaining variable X_3 with X_1 and X_2.

$$\alpha \frac{\text{cov}\,(X_1 - b_{13}\,X_3, X_2 - b_{23}\,X_3)}{\sqrt{\text{var}\,(X_1 - b_{13}\,X_3)\,\text{var}\,(X_2 - b_{23}\,X_3)}}$$

In which b_{13} and b_{23} are simple regression coefficient of X_1 on X_3 and X_2 on X_3 respectively. Then "α" is usually known as :

(a) Correlation ratio.

(b) Spurious correlation.

(c) Multiple correlation.

(d) Partial correlation.

27. Let (X_1, X_2, X_3) be three variables and let r_{ij} be the simple correlation co-efficient between these variables for $(i, j) = \{(1, 2), (1, 3), (2, 3)\}$. Let $r_{12}^2 + r_{13}^2 + r_{23}^2 - 2r_{12}\,r_{13}\,r_{23} = 1$, the regression planes : X_1 on X_2 and X_3 ; X_2 on X_1 and X_3 and X_3 on X_1 and X_2.

(a) Coincides.

(b) Not-coincides.

(c) Parallel.

(d) None of the above.

28. Short term fluctuations of a time series is called :

(a) Seasonal variation.

(b) Cyclic variation.

(c) Irregular variation.

(d) All the above.

29. Consider the decomposed time series model : $y_t = T_t + S_t C_t R_t$, this kind of model is known as :

(a) Additive model.

(b) Mixed model.

(c) Multiplicative model.

(d) None of the above.

30. Let $P_{oi}^{La} = a$ and $P_{oi}^{Pa} = b$ be the Laspeyre's and Paasche's Index respectively whereas a and b are its values. Then the Fisher's index number ($\left(P_{oi}^{F}\right)$ and Dorbish-Bowley Index number, $\left(P_{oi}^{DB}\right)$ are :

(a) $P_{oi}^{F} \sqrt{ab}$ and $P_{oi}^{DB} = \left(\frac{a-b}{2}\right)$.

(b) $P_{oi}^{F} \sqrt{a+b}$ and $P_{oi}^{DB} = \left(\frac{a+b}{2}\right)$.

(c) $P_{oi}^{F} \sqrt{ab}$ and $P_{oi}^{DB} = \left(\frac{a+b}{2}\right)$.

(d) $P_{oi}^{F} \sqrt{a+b}$ and $P_{oi}^{DB} = \left(\frac{a-b}{2}\right)$.

31. An index number is said to be time consistent if :

(a) $P_{oi} \times P_{io} = 1$.

(b) $P_{oi} \times Q_{io} = 1$.

(c) $P_{oi} \times P_{io} > 1$.

(d) $P_{oi} \times Q_{io} > 1$.

32. Find the correct statement :

Statement 1 : Tossing a biased coin is a random experiment.

Statement 2 : Getting a head while tossing a coin is called an event of the experiment.

(a) Statement 1 is right and statement 2 is wrong.

(b) Statement 1 is wrong and statement 2 is right.

(c) Both the statements are wrong.

(d) Both the statements are right.

33. If A and B are two events which are not disjoint, then P (A) + P (B) equals :

(a) $P(A \cup B)$.

(b) $P(A \cap B)$.

(c) $P(A \cup B) + P(A \cap B)$.

(d) $P(A \cup B) - P(A \cap B)$.

34. Let the cumulative distribution function of a continuous random variable X be F(x), then the cumulative distribution function of $Y = X + a$ is :

(a) $F(x + a)$. (b) $F(x - a)$.

(C) $F(2x)$. (d) None of the above.

35. Consider two events A and B, such that $P[A|B] = P(A)$ and $P(B) \neq 0$, then :

(a) A is statistically dependent on B.

(b) A is statistically independent on B.

(c) $P(A \cap B) = P(A \cup B)$.

(d) $P(A \cap B) = P(A \Delta B)$.

36. If an event B has occurred with probability one, then the conditional probability of A given B is :

(a) P (A). (b) P (B).

(c) 0. (d) 1.

37. If B is a subset of A then $P(A|B)$ is :

(a) P (A). (b) P (B).

(c) 0. (d) 1.

38. Given that $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{8}$ and $P(A|B) = \frac{1}{10}$, then the probability is $P(B|A)$ is :

(a) $\frac{1}{9}$. (b) $\frac{1}{16}$.

(c) $\frac{1}{19}$. (d) $\frac{1}{15}$.

39. Three coins are tossed simultaneously, the probability of getting at most one head is :

(a) 0. (b) 0.1.

(c) 0.5. (d) 1.1.

40. A random variable X has the following probability function :

Values of $X = x$	:	0	1	2	3	4	5	6
$P(x)$	:	0	k	$2k$	$4k$	$2k$	$8k^2$	$2k^2$

Find the value of k :

(a) 1.
(b) 2.
(c) 0.2.
(d) 0.1.

41. Let $F_{Y\backslash X}(y\backslash x)$ denotes the distribution function of Y when X has already assumed the value x. Then $F_{Y|X}(y\backslash x)$ can be written as :

(a) $1 - P(Y \leq y\backslash X = x)$.
(b) $1 - P(Y \geq y\backslash X = x)$.
(c) $1 - P(Y = y\backslash X = x)$.
(d) None of the above.

42. Let X be a continuous random variable with finite mean. Then its variance :

$$\int_{-\infty}^{+\infty} (X - E(X))^2 f(x)\, dx :$$

(a) Exists always.
(b) Never exist.
(c) Need not exists.
(d) Always finite.

43. Let X_1, X_2, X_3 be any 3 random variables with mean $E|X_1| = 3$, $E(X_2) = 7$, $E(X_3) = 3$. Then the mean of $5X_1 + 2|- X_2| + 20X_3$ is :

(a) 7.5.
(b) 75.
(c) 89.
(d) 8.9.

44. Consider the values of X : – 1, 2, – 3, 4, – 5, 6, – 7, 8, –9, 10, ..., 100, then find the correct relation :

(a) $E(|3X|) < |3E(X)|$.
(b) $E(|3X|) > |3E(X)|$.
(c) $E(|3X|) = |3E(X)|$.
(d) None of the above.

45. Let X and Y be any two dependent random variables. Then which among the given statement is true ?

Statement 1 : Cov (X, Y) = E (XY) – E (X) E (X) $\neq$ 0.

Statement 2: Var (X – Y) = Var (X) + Var (Y).

(a) Statement 1 is true and 2 is wrong.

(b) Statement 2 is true and 1 is wrong.

(c) Both statements are true.

(d) Both statements are wrong.

46. Let X and Y be two random variables. Consider the relation Min (X, Y) = $\frac{1}{2}$ [(X + Y) – (X – Y)] consider the value of X.

X	:	1	1	8	4	6
Y	:	2	1	9	8	10

Then :

(a) E [Min (X, Y)] $\leq$ Min [E (X), E (Y)].

(b) E [Min(X, Y)] > Min [E (X), E (Y)].

(b) E [Min(X, Y)] > 2 Min [E (X), E (Y)].

(d) None of the above.

47. Consider the experiment of tossing an unbiased coin 'n' times as a random experiment and assuming binomial distribution it will be :

(a) Asymmetric.

(b) Negatively skewed.

(c) Positively skewed.

(d) Symmetric.

48. Let X ~ Binomial (n, p), if , $n \to \infty$, $p \to 0$ and $np = \alpha$ then X follows :

(a) Binomial, B (n, α).

(b) Poisson, P (α).

(c) Negative Binomial, NB (n, α).

(d) Geometric, G (α).

49. Let X ~ N (99, σ^2) then E (X – 99) ^{99} is :

(a) 1.

(b) 0.

(c) 0.5.

(c) 9.9.

50. Let $X \sim N(0, 1)$ then $\int_0^{\infty} f(x)\,dx$ is equal to :

(a) 1.5.

(b) 1.

(c) 0.

(d) 0.5.

51. Consider a random variable X with probability density function.

$$f(x) = \frac{\left(\frac{1}{2}\right)^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)}\left[\exp\frac{(-x)}{2}\right]x^{\left(\frac{n}{2}-1\right)}, 0 < x < \infty.$$

Then the distribution of X is :

(a) Gamma $\left(\frac{1}{2}, \frac{n}{2}\right)$.

(b) Gamma $\left(\frac{n}{2}, \frac{n}{2}\right)$.

(c) Gamma $\left(\frac{1}{2}, \frac{1}{2}\right)$.

(d) $F\left(\frac{1}{2}, \frac{n}{2}\right)$.

52. Let X and Y be two independent standard normal variates. If $Y = \frac{X}{Y}$, then Y follows :

(a) Standard Normal distribution.

(b) Standard Cauchy distribution.

(c) Standard Laplace distribution.

(d) Chi-square distribution.

53. Identify the correct statement :

Statement 1 : A statistic is a function of the random sample.

Statement 2 : An estimator is a statistic that is used to estimate an unknown parameter θ of the distribution.

(a) Statement 1 is right and statement 2 is wrong.

(b) Statement 2 is right and statement 1 is wrong.

(c) Both statements are right.

(d) Both statements are wrong.

54. Consider an estimator $T_n = T(X_1, X_2, ..., Xn)$. If $E(T_n) = g(\theta)$, $Var(T_n) \neq g(\theta)$, for all $\theta \in \theta$. Then

(a) T_n is a minimal sufficient estimator of $g(\theta)$.

(b) T_n is consistent estimator of $g(\theta)$.

(c) T_n is a complete sufficient estimator of $g(\theta)$.

(d) T_n is an unbiased estimator of $g(\theta)$.

55. An estimator T_1 is said to be efficient than another estimator T_2 if :

(a) $Var(T_1) < Var(T_2)$. (b) $Var(T_1) > Var(T_2)$.

(c) $E(T_1) > Var(T_2)$. (d) $Var(T_1) < E(T_2)$.

56. Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$. Then the sufficient estimator for (μ, σ^2) is :

(a) $\left(\sum_{i=1}^{n} X_i^2, \sum_{i=1}^{n} X_i\right)$. (b) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i\right)$.

(c) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$. (d) $\left(\sum_{i=1}^{n} X_i^2, \sum_{i=1}^{n} X_i^2\right)$

57. Let T be an estimator for $g(\theta)$, then under the regularity conditions by Cramer-Rao inequality the lower bound of $Var(T)$ is :

(a) $\dfrac{\left\{\dfrac{d^2}{d\theta^2} g(\theta)\right\}}{E\left(\dfrac{\partial^2}{\partial\theta^2} \log L\right)}$. (b) $\dfrac{\left\{\dfrac{d^2}{d\theta^2} g(\theta)\right\}}{E\left(\dfrac{\partial}{\partial\theta} \log L\right)^2}$.

(c) $\dfrac{\left\{\dfrac{d^2}{d\theta} g(\theta)\right\}^2}{E\left(\dfrac{\partial}{\partial\theta} \log L\right)^2}$. (d) $\dfrac{\left\{\dfrac{d}{d\theta} g(\theta)\right\}^2}{E\left(\dfrac{\partial^2}{\partial\theta^2} \log L\right)}$.

58. Let 0, 1, 1, 0, 0, 0, 1, 2, 0, 1, 0, 0 be a random sample from a Poisson distribution with parameter λ. Now the moment estimator of λ is :

(a) 0.5.
(b) 1.5
(c) 1.
(d) 0.

59. Size of the critical region is known as :

(a) Power of the test.
(b) Size of type I error.
(c) Size of type II error.
(d) Size of the test.

60. Neyman-Pearson Lemma provides :

(a) An unbiased test.
(b) A most powerful test.
(c) Uniformly most powerful test.
(d) UMVUE.

61. Student's t Test was proposed by :

(a) C. R. Rao.
(b) Ronald A Fisher.
(c) P. C. Mahalanobis.
(d) William S. Gosset.

62. Degrees of freedom is related to :

(a) No. of observations in a set.
(b) Hypothesis under test.
(c) No. of independent observations in a set.
(d) None of the above.

63. Consider the random sample 2, 4, 6, 8 of size 4, then the sample variance is :

(a) $\frac{20}{3}$.
(b) $\frac{20}{6}$.
(c) 5.
(d) 1.

64. Errors in a survey other than sampling error is known as :

(a) Formula error.
(b) Non-sampling error.
(c) Planning error.
(d) None of the above.

65. A population is perfectly homogenous in respect of a characteristic. What sample of size will be more preferable ?

(a) A large sample.
(b) A small sample.
(c) A single item.
(d) No item.

66. Randomization is a process in which treatments are allocated to the experimental units :

(a) In a sequence.
(b) By the convenience of the investigator.
(c) With equal probability.
(d) None of the above.

67. Replication in an experiment eliminates :

(a) Human bias.
(b) Competition among neighbouring plots.
(c) Heterogeneity among groups.
(d) None of the above.

68. Control charts in statistical quality control are meant for :

(a) Describing the pattern of variation.
(b) Checking whether the variability in the product is within the tolerance limits or not.
(c) Uncovering whether the variability in the product is due to assignable causes or not.
(d) All the above.

69. Main tools of statistical quality control are :

(a) Shewart charts.
(b) Acceptance sampling plan.
(c) Both (a) and (b).
(d) None of the above.

70. Shewart control chart is meant to :

(a) Detect whether the process is under statistical quality control.

(b) Find the assignable causes.

(c) Reflect the selection of samples.

(d) All the above.

71. A defect in an item is classified as major if :

(a) It stops the function of the process.

(b) It is not detectable.

(c) It shortens the life of the system.

(d) All the above.

72. The control limit delimited by the consumer is called :

(a) Modified control limit.

(b) Natural control limit.

(c) Specified control limit.

(d) None of the above.

73. The death rate of children under one month is known as :

(a) Neonatal mortality rate.

(b) Infant mortality rate.

(c) Maternal mortality rate.

(d) Foetal death rate.

74. In a linear programming problem dual of a dual is :

(a) Dual.

(b) Primal.

(c) Either dual or primal.

(d) neither dual nor primal

75. Simplex method was proposed by :

(a) Abraham Wald.

(b) G. Taguchi.

(c) G. B. Dantzig.

(d) Bradley Efron.