D 130227 **(Pages : 3)** **Name..**

Reg. No......................................

FIFTH SEMESTER CBCSS–UG DEGREE EXAMINATION NOVEMBER 2025

(2020 Syllabus)

Mathematics

MTS 5B 06—BASIC ANALYSIS

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Short Answer Type
All questions can be answered.
Each question carries 2 Marks. (Ceiling 25 Marks)

1. Let $a \in \mathbb{R}$. Prove that $a.0 = 0$

2. Let $a \in \mathbb{R}$. with $a \neq 0$. Prove that $a^2 > 0$.

3. Let x, y belongs to the e-neighbourhood of a, b. Prove that $x + y$ belongs to the 2e-neighbourhood of $a + b$.

4. Prove that an upper bound of u of a non-empty set S in $\mathbb{R}$ is the suprimum of S if and only if for every $\in > 0$ there exist an $s_{\in} \in \mathrm{S}$ such that $u - \in < s_{\in}$.

5. Let S be a non-empty subset of $\mathbb{R}$ that is bounded above and let $a \in \mathbb{R}$. Prove that $\sup(a+s) = a + \sup \mathrm{S}$.

6. Consider the sequences $\mathrm{X} = (2, 4, 6, \ldots, 2n, \ldots)$ and $\mathrm{Y} = \left(\frac{1}{1}, \frac{1}{2}, \frac{1}{8}, \ldots, \frac{1}{n} \ldots\right)$. Find XY and X/Y.

7. Let (x_n) be a sequence of real numbers that converges to x. Prove that $(|x_n|)$ converges to $|x|$.

8. Let (x_n) be a sequence of real numbers that converges to x. Prove that any subsequence $\left(x_{n_k}\right)$ of (x_n) also converges to x.

Turn over

9. Whether the sequence $\left(1+(-1)^{n}\right)$ is a Cauchy sequence ? Justify your answer.

10. Let $c > 1$, find the limit $\lim(c^{n})$.

11. Find the modulus of the complex number $\dfrac{2i}{3-4i}$.

12. Find the polar form of the complex number $-\sqrt{3}-i$.

13. Express the number $3i^{5}-i^{4}+7i^{3}-10i^{2}-9$ in the form of $a + ib$.

14. Sketch the graph of $Re(z)=5$ in the complex plane.

15. Find the real and imaginary parts u and v of the complex function $f(z)=z^{3}-2z+6$.

(Ceiling 25 marks)

Section B

Paragraph / Problem Type.
All questions can be answered.
Each question carry 5 marks.
(Ceiling 35 marks).

16. Let A be any set. Prove that there is no surjection of A onto the set $\mathcal{P}(\mathrm{A})$ of all subsets of A.

17. Let $a,b\in\mathbb{R}$. Prove that (i) $|ab|=|a||b|$. (ii) $|a+b|\le|a|+|b|$.

18. Let $x,y\in\mathbb{R}$ with $x<y$. Prove that there exist an irrational number z such that $x<z<y$.

19. Let $\mathrm{X}=(x_{n})$ be a sequence of real numbers that converges to x and $\mathrm{Z}=(z_{n})$ be a sequence of non-zero real numbers that converges to z with $z\neq 0$. Prove that X/Z converges to x/z.

20. Let (x_{n}) be a bounded increasing sequence. Prove that $\lim(x_{n})=\sup\{x_{n}:n\in\mathbb{N}\}$.

21. Find $\text{Re}(z)$ and $\text{Im}(z)$ where $z = \left(\frac{i}{3-i}\right)\left(\frac{1}{2+3i}\right)$.

22. Describe the set of points z in the complex plane satisfy $|z-i| = |z-1|$.

23. Prove that the union of an arbitrary collection of open subsets in $\mathbb{R}$ is open.

(Ceiling 35 marks)

Section C

Essay type.
Answer any **two** *of the following questions.*
Each question carries 10 marks.

24. Prove that the set $\mathbb{R}$ of all real numbers is uncountable.

25. State and prove Nested Intervals Property.

26. Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Prove that the sequence $X = \left(\sqrt{x_n}\right)$ of positive square roots converges to $\sqrt{x}$.

27. Find the image of the vertical line $x = 1$ under the complex mapping $w = z^2$.

(2 × 10 = 20 marks)