D 93427 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 05—NUMBER THEORY

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

General Instructions.

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Find all integers n such that :

   $\phi(n) = n/2$.

2. If $n \geq 1$, prove that :

   $$\wedge(n) = \sum_{d/n} \mu(d) \cdot \log \frac{n}{d}.$$

3. Prove that $[2x] - 2[x]$ is either 0 or 1.

4. For $x > 0$, prove that :

   $$0 \leq \frac{\psi(x)}{x} - \frac{\vartheta(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x}\log 2}.$$

5. For all $x \geq 1$, prove that :

   $$\sum_{p \leq x} \frac{\log p}{p} = \log x + O(1).$$

6. Evaluate the Legendre' symbol $(73 \mid 383)$.
7. Prove that Legendre's symbol $(n \mid p)$ is a completely multiplicative function n of n.
8. Briefly describe about digraph transformation.

($8 \times 1 = 8$ weightage)

Turn over

Part B

Answer any **six** *questions by choosing two questions from each unit.*
Each question carries a weightage of 2.

Unit I

9. For $n \geq 1$, prove that :

$$\phi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right).$$

10. State and prove the selberg identity.

11. If $x \geq 1$, prove that :

$$\sum_{n>x} \frac{1}{n^s} = O\left(x^{1-s}\right) \text{ if } s > 1.$$

Unit II

12. Prove that the relation :

$$\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$$

implies the relation :

$$\lim_{x \to \infty} \frac{\vartheta(x)}{x} = 1.$$

13. Let $\{a(n)\}$ be a non-negative sequence such that :

$$\sum_{n \leq x} a(n) \left[\frac{x}{n}\right] = x \log x + O(x) \text{ for all } x \geq 1.$$

For $x \geq 1$, prove that :

$$\sum_{n \leq x} \frac{a(n)}{n} = \log x + O(1).$$

14. State and prove the Abel's identity.

Unit III

15. For every odd prime p, prove that :

$$\left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8}.$$

16. Describe briefly about RSA cryptosystem.

17. In the 27-letter alphabet (with blank = 26), use the affine enciphering transformation with key $a = 13, b = 9$ to encipher the message "HELP ME".

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. Show that the set of all arithmetical functions f with $f(1) \neq 0$ forms an abelian group with respect to the Dirichlet product.

19. For every integer $n \geq 2$, prove that :

$$\frac{1}{6}\frac{n}{\log n} < \pi(n) < 6\frac{n}{\log n}.$$

20. If p and q are distinct odd primes, then prove that :

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{(p-1)(q-1)/4}.$$

21. The message "KVW ? Ta!KJB ? FVR •" (The blanks after ? and R are part of the message, but the final • is not) is intercepted. It is known that a linear enciphering transformation is being used with a 30-letter alphabet, in which A – Z have numerical equivalents 0 – 25, blank = 26, ? = 27, ! = 28, • = 29. It is further known that the first six letters of the plaintext are "C.I.A". Find the deciphering matrix A^{-1} and the full plaintext message.

(2 × 5 = 10 weightage)