

**THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2020**

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all the twelve questions.

Each question carries 1 mark.

1. What do you mean by a homogeneous equation ?
2. Consider a system of linear equations in n unknowns with augmented matrix $M = [A, B]$. Then, the solution is unique if and only if rank (A).
3. What is the order of the differential equation $y \left(\frac{dy}{dx} \right)^2 + 8x = 0$.
4. State Cayley-Hamilton theorem.
5. What is the determinant of a 2×2 matrix whose rank is 1 ?
6. What is the normal form of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{pmatrix}$?
7. Define eigen value of a matrix.
8. Define divergence of a vector field.
9. Define gradient of a function.
10. Define the derivative of a vector function.
11. Define a smooth curve.
12. State Gauss's divergence theorem.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** questions.

Each question carries 2 marks.

13. Solve the initial value problem $y' = 3y$, $y(0) = 5.7$.

14. Find an integrating factor for $2\cosh x \cos y dx = \sinh x \sin y dy$ and solve it.

15. Find the angles of the triangle with vertices $(0, 0, 0)$, $(1, 2, 3)$, $(4, -1, 3)$.

16. Find x, y, z, t where $3 \begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix}$.

17. Solve the system :

$$x - 3y = 4$$

$$-2x + 6y = -8.$$

18. Show that circle of radius a has curvature $\frac{1}{a}$.

19. Find a unit normal vector n of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point $(1, 0, 2)$.

20. Find the directional derivative of $f = x^2 + y^2 - z$ at $(1, 1, -2)$ in the direction of $(1, 1, 2)$.

21. Show that $\text{curl}(u + v) = \text{curl } u + \text{curl } v$.

22. Show that $\text{div } kv = k \text{div } v$.

23. Show that $\int_{(0, \pi)}^{(3, \frac{\pi}{2})} e^x (\cos y dx - \sin y dy)$ is path independent.

24. Write the formula for finding the area of a plane region as a line integral over the boundary.

(9 × 2 = 18 marks)

Part C (Short Essay)

Answer any six questions.

Each question carries 5 marks.

25. Show that the form under integral sign is exact in the plane and evaluate the integral

$$\int_{(-1,-1)}^{(1,1)} e^{-x^2-y^2} (xdx + ydy).$$

26. Solve $2x \tan y dx + \sec^2 y dy = 0$.

27. Find the minimal polynomial $m(t)$ of $A = \begin{pmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{pmatrix}$.

28. Let $A = \begin{pmatrix} 3 & -4 \\ 2 & -6 \end{pmatrix}$. Find all eigen values and corresponding eigen vectors. Find matrices P and D such that P is non-singular and $D = P^{-1}AP$ is diagonal.

29. Let L be the linear transformation on \mathbb{R}^2 that reflects each point P across the line $y = kx$, where $k > 0$.

(a) Show that $v_1 = (k, 1)$ and $v_2 = (1, -k)$ are eigenvectors of L.

(b) Show that L is diagonalizable, and find a diagonal representation D.

30. Find the straight line L_1 through the point $P : (1, 3)$ in the xy -plane and perpendicular to the straight line $L_2 : x - 2y + 2 = 0$.

31. Evaluate the double integral $\iint_R y^2 dx dy$ where R is the region bounded by the unit circle in the first quadrant.

32. Solve $2x \tan y dx + \sec^2 y dy = 0$.

33. Verify Greens theorem in the plane for $F = [-y^3, x^3]$ and the region is the circle $x^2 + y^2 = 25$.

(6 × 5 = 30 marks)

Turn over

Part D

Answer any two questions.

Each question carries 10 marks.

34. Test for consistency and solve the following system :

$$\begin{aligned} \text{(a)} \quad & x_1 + x_2 - 2x_3 + 4x_4 = 5 \\ & 2x_1 + 2x_2 - 3x_3 + x_4 = 3 \\ & 3x_1 + 3x_2 - 4x_3 - 2x_4 = 1. \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & x + 2y + z = 3 \\ & 2x + 5y - z = -4 \\ & 3x - 2y - z = 5. \end{aligned}$$

35. Solve :

$$\text{(a)} \quad 2 \sin(y^2) dx + xy \cos(y^2) dy = 0, y(2) = \sqrt{\frac{\pi}{2}}.$$

(b) Find the angle between $x - y = 1$ and $x - 2y = -1$.

36. Evaluate $\iint_S (7xi - zk) \cdot ndA$ over the sphere $S: x^2 + y^2 + z^2 = 4$ by

(a) Divergence theorem.

(b) Directly.

(2 × 10 = 20 marks)