C 21827 (Pages : 4) Name.....

Reg. No.....

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2022

Mathematics

MAT 2C 02—MATHEMATICS

(2016—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Find the n^{th} term of the sequence 2, 5, 10, _____.

2. $\frac{d}{dx}(\tanh x) =$ _____.

3. If f is continuous on $[a, \infty)$, then $\lim_{b \to \infty} \int_a^b f(x)\,dx =$ _____.

4. The n^{th} term of a converging infinite series has the limit _____.

5. Find the domain of the function $w = \sin xy$.

6. Define level curve of a function f.

7. State Euler's mixed derivative theorem.

8. If $f(x, y) = \cosh xy$, find $\frac{\partial}{\partial x} f(x, y)$.

9. Which point is the foot of the perpendicular from the origin to the line $r\cos(\theta - \theta_0) = r_0$?

10. The name of the curve with the polar equation $r = a(1 + \cos\theta)$ is _____.

Turn over

11. Define partial derivative of $u = f(x, y, z)$ with respect to y.

12. State the chain rule for partial differentiation in the case of functions with two independent variables and three intermediate variables.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Investigate the convergence of $\int_0^1 \frac{1}{1-x}\,dx$.

14. The region between the curve $y = \sqrt{x}, 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

15. Determine the sequence $a_n = \frac{3n-1}{n+1}$ is nondecreasing and if it is bounded from above.

16. Find the cartesian form of the curve $r = 2a \sin\theta$.

17. Find $\frac{\partial}{\partial z}$ if the equation $yz - \ln z = xy$ defines z as a function of the two independent variables x and y and the partial derivatives exists.

18. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

19. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = x^2 + y^2 - 2z^2$.

20. Investigate the convergence of $\int_1^\infty \frac{\sin^2 x}{x^2}$.

21. Find the sum of the series : $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots.$

22. Find the Maclaurin series expansion of $f(x)=(1+x)^n$.

23. Find $f_x(1,2)$ if $f(x,y)=\cosh^2(xy)-\sinh^2(xy)$.

24. Test the function for continuity at origin :

$$f(x,y)=\begin{cases}\frac{2xy}{x+y^2}, & \text{if } (x,y)\neq(0,0),\\ 0, & \text{otherwise}\end{cases}.$$

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions*
Each question carries 5 *marks.*

25. Show that $\tanh(2x)=\frac{2\tanh x}{1+\tanh^2 x}$.

26. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$ and find the actual value in case of convergence

27. Find the sum of the series $\sum_{n=1}^{\infty}\frac{3^{n-1}-1}{6^n}$.

28. Find the linearization of $f(x,y)=x^2-xy+\frac{1}{2}y^2+3$ at the point $(3,2)$.

29. Find the length of the cardioid $r=2(1-\cos\theta)$.

30. Use partial differentiation to find $\frac{dx}{dy}$ if $x^3+y^3-3xy=0$.

Turn over

31. Find the cartesian form of the surface $\rho = 2\cos\theta$.

32. Find the radius and centre of the circle $r = 4\cos\theta$ in cartesian plane.

33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (2x-1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions*
Each question carries 10 *marks.*

34. If the resistors R_1, R_2, and R_3 hms are connected in parallel to make an R ohm resistor satisfying the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ find $\frac{\partial R}{\partial R_2}$ when $R_1 = 30, R_2 = 45$ and $R_3 = 90$ ohms.

35. (a) Find a curve through the point $(1, 1)$ whose length integral is $L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx$.

(b) How many such curves axe there ? Give reasons for your answer.

36. Find the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le \frac{1}{2}$ about the x-axis.

(2 × 10 = 20 marks)