3403

D 93424 (Pages : 3) Name..................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 02—LINEAR ALGEBRA

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *the questions.*
Each question carries 1 *weightage.*

1. Show that if V is a finite-dimensional vector space, then any two bases of V have the same number of elements.
2. Verify that the function T from R^2 into R^2 defined by $T(x_1,x_2)=(x_1-x_2,0)$ is a linear transformation.
3. Let T be the linear operator on C^2 defined by $T(x_1,x_2)=(x_1,0)$ and let $B=\{(1,0),(0,1)\}$ and $B'=\{(1,i),(-i,2)\}$ be two ordered bases for C^2. What is the matrix of T relative to the pair B,B'?
4. Show that if S is any subset of a finite-dimensional vector space V, then $(S^o)^o$ is the subspace spanned by S.
5. Let T be the linear operator on R^2, the matrix of which in the standard ordered basis is $A=\begin{bmatrix}1 & -1\\ 2 & 2\end{bmatrix}$. prove that the only subspaces of R^2 invariant under T are R^2 and the zero space.

Turn over

. Prove that if E is the projection on R along N, then (I – E) is the projection on N along R.

7. Define inner product on a vector space and show that if $(\alpha \mid \beta) = 0$ for all β in an inner product space V, then $\alpha = 0$.

8. Consider C, with the standard inner product. Find an orthonormal basis for the subspace spanned by $\beta_1 = (1,0,i)$ and $\beta_2 = (2,1,1+i)$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each of the following units.*
Each question carries 2 weightage.

Unit I

9. Show that a non-empty subset W of a vector space V is a subspace of V iff for each pair of vectors α, β in W and each scalar c in the scalar field F the vector $c\alpha + \beta$ is in W.

10. Find the co-ordinate matrix of the vector (1, 0, 1) in the basis of c^3 consisting of the vectors $(2i,1,0), (2,-1,1), (0,1+i,1-i)$ in that order.

11. Let T be the linear operator on R^3 defined by $T(x_1,x_2,x_3),(3x_1,x,-x_2,2x_1+x_2+x_3)$. Is T invertible? If so, find a rule for T^{-1}.

Unit II

12. Let V be a finite-dimensional vector space over the field F, and let $B = \{\alpha_1, \dots, \alpha_n\}$ be a basis for V. Show that there is a unique dual basis $B^* = \{f_1, \dots, f_n\}$ for the dual space V^*.

13. Let V and W be vector spaces over the field F, and let T be a linear transformation from V into W. Show that the null space of T^t is the annihilator of the range of T. Show further that if V and W are finite-dimensional then rank (T^t) = rank (T).

14. Let T be a diagonalizable linear operator on an n-dimensional vector space V, and let W be an invariant subspace under T. Prove that the restriction operator T_w is diagonalizable.

Unit III

15. Let V be a finite-dimensional vector space and let w_1 be any subspace of V. Show that there is a subspace w_2 of V such that $V = w_1 \oplus w_2$.

16. Let E be a projection of V and let T be a linear operator on V. Prove that the range of E is invariant under T iff ETE = TE.

17. Show that an orthogonal set of non-zero vectors is linearly independent in an inner product space.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 weightage.

18. If w_1 and w_2 are finite-dimensional subspaces of a vector space V, then show that :

    (i) $w_1 + w_2$ is a finite-dimensional subspace of V.

    (ii) $\dim w_1 + \dim w_2 = \dim (w_1 \cap w_2) + \dim (w_1 + w_2)$

19. Let W be the subspace of R^5 which is spanned by the vectors $\alpha_1 = (1,2,1,0,0)$, $\alpha_2 = (0,1,3,3,1)$, $\alpha_3 = (1,4,6,4,1)$. Find a basis for w^o.

20. Let T be a linear operator on a finite dimensional vector space V. Prove that if f is the characteristic polynomial for T, then $f(T) = 0$.

21. Let W be a finite-dimensional subspace of an inner product space V and let E be the projection of V on W. Show that :

    (i) E is an idempotent linear transformation of V onto W.

    (ii) $W^{\perp}$ is the null space of E.

    (iii) $V = W \oplus W^{\perp}$.

(2 × 5 = 10 weightage)