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(Pages : 4)

Name.....

Reg. No.....

**FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY)  
EXAMINATION, NOVEMBER 2025**

(CBCSS)

Mathematics

MTH1C03—REAL ANALYSIS—I

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

**Part A (Short Answer Type Questions)***Answer all questions.**Each question carries a weightage 1.*

1. Examine whether the following subset of  $\mathbb{R}^2$  is : (i) Closed ; (ii) Open (iii) Perfect ; and (iii) bounded.

The set of all complex numbers  $z$  such that  $|z| \leq 1$ .

2. If  $X$  is a metric space and  $E \subset X$ , then prove that  $\bar{E} \subset F$  for every closed set  $F \subset X$  such that  $E \subset F$ .
3. Discuss the continuity/discontinuity behaviour of the function  $f$  defined by :

$$f(x) = \begin{cases} x+2 & (-3 < x < -2) \\ -x-2 & (-2 \leq x < 0) \\ x+2 & (0 \leq x < 1) \end{cases}$$

4. Define separated subsets of a metric space.
5. Suppose  $f$  is differentiable in  $(a, b)$  and  $f'(x) \leq 0$  for all  $x \in (a, b)$ , then prove that  $f$  is monotonically decreasing.
6. If  $f \in \mathfrak{R}(\alpha)$  and  $g \in \mathfrak{R}(\alpha)$  on  $[a, b]$ , then prove that  $fg \in \mathfrak{R}(\alpha)$

Turn over

7. Define equicontinuous family of functions.  
8. Define rectifiable curve.

(8 × 1 = 8 weightage)

**Part B (Paragraph Type Questions)***Answer any two questions from each module.**Each question carries a weightage 2.*

## MODULE I

9. If  $\{I_n\}$  is a sequence of intervals in  $\mathbb{R}^1$ , such that  $I_n \supset I_{n+1}$  ( $n = 1, 2, 3, \dots$ ), then prove that

$$\bigcap_{n=1}^{\infty} I_n \text{ is not empty.}$$

10. Prove that every  $k$ -cell is compact.  
11. Suppose  $f$  is a continuous mapping of a compact metric space  $X$  into a metric space  $Y$ . Then prove that  $f(X)$  is compact.

## MODULE II

12. Let  $f$  be defined by :

$$f(x) = \begin{cases} x \sin \frac{1}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$$

Find  $f'(x)$  (for  $x \neq 0$ ). Also prove that  $f'(0)$  does not exist.

13. If  $P^*$  is a refinement of  $P$ , then prove that :

$$L(P, f, \alpha) \leq L(P^*, f, \alpha)$$

14. If  $f_1 \in \mathfrak{R}(\alpha)$  and  $f_2 \in \mathfrak{R}(\alpha)$  on  $[a, b]$ , then prove that  $f_1 + f_2 \in \mathfrak{R}(\alpha)$ , and

$$\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha.$$

## MODULE II

15. If  $\gamma'$  is continuous on  $[a, b]$ , then prove that  $\gamma$  is rectifiable, and

$$\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt.$$

16. Give an example of an everywhere discontinuous limit function, which is not Riemann-integrable.
17. If  $K$  is compact, if  $f_n \in C(K)$  for  $n = 1, 2, 3, \dots$ , and  $\{f_n\}$  is pointwise bounded and equicontinuous on  $K$ , then prove that  $\{f_n\}$  is uniformly bounded on  $K$ .

(6 × 2 = 12 weightage)

**Part C (Essay Type Questions)**

*Answer any two questions.*

*Each question carries a weightage 5.*

18. Let  $\{E_n\}$ ,  $n = 1, 2, 3, \dots$ , be a sequence of countable sets, and put

$$S = \bigcup_{n=1}^{\infty} E_n.$$

Then prove that  $S$  is countable.

19. Suppose  $f$  is a real function on  $[a, b]$ ,  $n$  is a positive integer,  $f^{(n-1)}$  is continuous on  $[a, b]$ ,  $f^{(n-1)}(t)$  exists for any  $t \in (a, b)$ . Let  $\alpha, \beta$  be distinct points of  $[a, b]$ , and define

$$P(t) = \sum_{k=0}^{n-1} \frac{f^{(k)}(\alpha)}{k!} (t - \alpha)^k.$$

Then prove that there exists a point  $x$  between  $\alpha$  and  $\beta$  such that :

$$f(\beta) = P(\beta) + \frac{f^{(n)}(x)}{n!} (\beta - \alpha)^n.$$

**Turn over**

20. Prove that  $f \in \mathcal{R}(\alpha)$  on  $[a, b]$  if and only if for every  $\varepsilon > 0$  there exists a partition  $P$  such that :

$$U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon.$$

21. Let  $\alpha$  be monotonically increasing on  $[a, b]$ . Suppose  $f_n \in \mathcal{R}(\alpha)$  on  $[a, b]$ , for  $n = 1, 2, 3$  and suppose  $f_n \rightarrow f$  uniformly on  $[a, b]$ . Then prove that  $f \in \mathcal{R}(\alpha)$  on  $[a, b]$ , and

$$\int_a^b f d\alpha = \lim_{n \rightarrow \infty} \int_a^b f_n d\alpha.$$

(2 × 5 = 10 weightage)