C 32336 (Pages : 3) Name..............................

Reg. No..............................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Each question carries weightage 1.

1. Let v be a vector space over a field F. Show that if c is a scalar and o is the zero-vector then c.o = 0.

2. Describe all subspaces of R^2.

3. Let W_1 and W_2 be subspaces of a vector space V such that $W_1 + W_2 = V$ and $W_1 \cap W_2 = \{0\}$. Prove that for each vector α in V there are unique vectors α_1 in W_1 and α_2 in W_2 such that $\alpha = \alpha_1 + \alpha_2$.

4. Write a basis for the vector space of all 2×2 matrices over the field F of real numbers. Justify your answer.

5. Verify that the function $T : R^2 \to R^2$ defined by $T(x_1, x_2) = (1 + x_1, x_2)$ is a linear transformation.

6. Let T be the linear transformation from R^3 into R^2 defined by $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$. If $\mathscr{B}$ is the standard ordered basis for R^3 and $\mathscr{B}'$ is the standard ordered basis for R^2, What is the matrix of T relative to $\mathscr{B}$ and $\mathscr{B}'$.

7. Show that if s is a subset of a vector space V, then s^o is a subspace of V^*.

8. Show that similar matrices have the same characteristic polynomial.

9. Let V be a finite-dimensional vector space. Determine the minimal polynomial for a diagonal operator on V.

10. Let E be a projection of a vector space with range R. Show that a vector β is in the range R iff $\beta = E\beta$.

Turn over

11. Let T be the linear operator on R^2, the matrix of which in the standard ordered basis is $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$.

Show that the subspace W_1 spanned by the vector (1, 0) is invariant under T.

12. Define inner product on a vector space.

13. Show that if s is any subset of a vector space V, then its orthogonal complement $s^{\perp}$ is a subspace of V.

14. Let (|) be the standard inner product on R^2. Let $\alpha = (1, 2),\ \beta = (-1, 1)$. If γ is a vector in R^2 such that $(\alpha/\gamma) = -1$ and $(\beta/\gamma) = 3$, find γ.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Let V be a vector space over the field F. Show that the intersection of any collection of subspaces of V is a subspace of V.

16. Let $\mathscr{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis for R^3 consisting of $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1)$, $\alpha_3 = (1, 0, 0)$. What are the co-ordinates of the vector (a, b, c) in the ordered basis $\mathscr{B}$?

17. Let T be a linear transformation from a vector space V into a vector space W over the same field F. Show that T is non-singular iff T carries each linearly independent subset of V onto a linearly independent subset of W.

18. Let V be a finite-dimensional vector space over the field F, and let W be a subspace of V. Show that $\dim W + \dim W^{o} = \dim V$.

19. Let T be a linear operator on an n-dimensional vector space V. Show that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.

20. Let T be a diagonalizable linear operator on the n-dimensional vector space V, and let W be a subspace which is invariant under T. Prove that the restriction operator T_W is diagonalizable.

21. Let V be the vector space of all $n \times n$ matrices over R. Let $W_1 = \{A \varepsilon V : A^t = A\}$ and $W_2 = \{A \varepsilon V : A^t = -A\}$ be subspaces of V. Show that $W_1 \oplus W_2 = V$.

22. Let E be a projection and T be a linear operator on a vector space V. show that the range of E is invariant under T iff ETE = TE.

23. Show that if V is an inner product space, then for any vectors α, β in V ; $|(\alpha|\beta)| \leq \|\alpha\| \cdot \|\beta\|$.

24. State and prove Bessel's inequality.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. Show that if W_1 and W_2 are finite dimensional subspaces of V, then $W_1 + W_2$ is finite-dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$.

26. Let W be the subspace of R^5 spanned by the vectors $\alpha_1 = (2, -2, 3, 4, -1)$, $\alpha_2 = (-1, 1, 2, 5, 2)$, $\alpha_3 = (0, 0, -1, -2, 3)$, $\alpha_4 = (1, -1, 2, 3, 0)$. Describe W^o, the annihilator of W.

27. Let V be a finite-dimensional vector space over the field F and Let T be a linear operator on V. Show that T is triangulable iff the minimal polynomial for T is a product of linear polynomials over F.

28. Apply the Gram-Schmidt process to the vectors $\beta_1 = (1, 0, 1)$, $\beta_2 = (1, 0, -1)$, $\beta_3 = (0, 3, 4)$, to obtain an orthonormal basis for R^3 with the standard inner product.

(2 × 4 = 8 weightage)