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| Q.P Code   | D 113035 | Total Pages: 2 | Name              |
|  |          |                | Register No.      |
| <b>FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2024</b> |          |                |                   |
| <b>(CUFYUGP)</b>   |          |                |                   |
| <b>MAT1MN102 - CALCULUS OF A SINGLE VARIABLE</b>           |          |                |                   |
| <b>2024 Admissions</b>                                     |          |                |                   |
| Maximum Time :2 Hours                                      |          |                | Maximum Marks :70 |

### Section A

All Question can be answered. Each Question carries 3 marks (Ceiling : 24 Marks)

|    |  |
|----|--|
| 1  | Find $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x - 3}$   |
| 2  | Find $\lim_{x \rightarrow 2^+} \frac{x}{x^2 - 4}$  |
| 3  | Does the function $f(x) = \begin{cases} \sin x & \text{if } x \leq \pi/2 \\ x^2 & \text{if } x > \pi/2 \end{cases}$ , continuous at $\pi/2$ ? Why?   |
| 4  | A robot moves in the positive direction along a straight line so that after t minutes its distance is $s = 6t^4$ feet from the origin. Find the average velocity of the robot over the interval $[2, 4]$ . |
| 5  | Using the definition of differentiation find $\frac{d}{dx}[x^2]$   |
| 6  | Find $\frac{dy}{dx}$ if $y = \ln(\ln(\ln x))$  |
| 7  | Find $\lim_{x \rightarrow 0} \frac{\ln \cos x}{x}$ ; $x \in (0, \pi/2)$  |
| 8  | Show that the function $f(x) = x^4 - 2x^3$ is not one-to- one on $(-\infty, \infty)$ .   |
| 9  | Explain the term concavity of a function   |
| 10 | Find all critical points of $f(x) = 3x^{5/3} - 15x^{2/3}$  |

## Section B

**All Question can be answered. Each Question carries 6 marks (Ceiling : 36 Marks))**

|    |   |
|----|---|
| 11 | $\lim_{x \rightarrow +\infty} \frac{1 - e^x}{1 + e^x}$  |
| 12 | Discuss the continuity of the function $\frac{x+2}{x^2-4}$  |
| 13 | Let $f(x) = \begin{cases} x^2 - 3x + 2 & \text{if } x \leq 2 \\ x & \text{if } x > 2 \end{cases}$<br>Determine whether $f$ is differentiable at $x = 2$ . If so, find the value of the derivative there |
| 14 | Find $\frac{d^2y}{dx^2}$ , if $y = \sin \sqrt{x} + \cos(5x)$  |
| 15 | At what point is the tangent line to the curve $y^3 = 2x^2$ perpendicular to the line $x + 2y - 2 = 0$ ?  |
| 16 | Use logarithmic differentiation to find $\frac{dy}{dx}$ , if $y = (x^2 + 1)^{\cos x}$   |
| 17 | Find the inflection points of the function $f(x) = xe^{-x}$   |
| 18 | Find the relative extrema of $f(x) = \frac{x+3}{x-2}$   |

## Section C

**Answer any ONE. Each Question carries 10 marks (1x10=10 Marks))**

|    |  |
|----|--|
| 19 | Using the definition of differentiation find<br><br>1. $\frac{d}{dx}[\sin x]$<br><br>2. $\frac{d}{dx}[\sqrt{x}]$   |
| 20 | Consider the function $f(x) = x^5 + x + 1$ .<br><br>1. Show that $f$ is one-to-one on the interval $(-\infty, \infty)$ .<br><br>2. Find a formula for the derivative of $f$ .<br><br>3. Compute $(f^{-1})'(1)$ |