

D 93423

(Pages : 2)

Name.....

Reg. No.....

**FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY)
EXAMINATION, NOVEMBER 2020**

(CBCSS)

Mathematics

MTH 1C 01—ALGEBRA I

(2019 Admissions)

Time : Three Hours

Maximum : 30 Weightage

Part A

*Answer all questions.
Each question has weightage 1.*

1. Verify whether $\phi(x, y) = (x + 1, y)$ is an isometry of the plane.
2. Find a generator of the cyclic group $\mathbb{Z}_2 \times \mathbb{Z}_3$.
3. Describe all abelian groups of order 20 upto isomorphism.
4. Find all subgroups of the quotient group $\mathbb{Z}/6\mathbb{Z}$.
5. Let G be a group of order 18. Find the number of 3-Sylow subgroups of G .
6. Give all elements of the group given by the presentation $(a : a^5 = 1)$.
7. Find the inverse of $2i + j + k$ in the ring of quaternions.
8. Let $\phi : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ be the homomorphism defined by $(x, y) \mapsto (x, 0)$. Find $\ker \phi$.

(8 × 1 = 8 weightage)

Part B

*Answer any six questions, choosing two from each unit.
Each question has weightage 2.*

Unit 1

9. Verify whether $\mathbb{Z}_5 \times \mathbb{Z}_6$ is a cyclic group.
10. Show that $\mathbb{Z}/5\mathbb{Z}$ is isomorphic to \mathbb{Z}_5 .
11. Verify whether \mathbb{Z}_6 is simple.

Turn over

Unit 2

12. Show that $(1\ 2\ 3)$ and $(1\ 3\ 2)$ are conjugates in the symmetric group S_3 .
13. Verify whether the following series of groups are isomorphic :

$$0 < (5) < \mathbb{Z}_{15} \text{ and } 0 < (3) < \mathbb{Z}_{15}.$$

14. Show that every group of order 15 is cyclic.

Unit 3

15. Show that the ring of all endomorphisms of the group \mathbb{Z} of integers is commutative.

16. Let $\phi_2 : \mathbb{Q}[x] \rightarrow \mathbb{Q}$ be the evaluation homomorphism with $\phi_2(x) = 2$. Find the kernel of ϕ_2 .

17. Verify whether $x^3 + 3x + 2 \in \mathbb{Z}_5[x]$ is irreducible.

(6 × 2 = 12 weightage)

Part C

*Answer any two questions.
Each question has weightage 5.*

18. (a) Let G be a group and H be a normal subgroup of G .
- Show that the coset multiplication $(aH)(bH) = (ab)H$ is well defined.
 - Verify that G/H is a group.
- (b) Describe the factor group \mathbb{Z}_{20}/H where H is the subgroup generated by 5.
19. (a) Let H, K be groups and $G = H \times K$. Show that :
- $\bar{H} = \{(h, e) : h \in H\}$ is a normal subgroup of G where e is the identity of K .
 - G/\bar{H} is isomorphic to K .
- (b) Show that factor groups of cyclic groups are cyclic.
20. Let G be a group and N be a normal subgroup of G and H be any subgroup of G . Show that :
- $HN = NH$.
 - HN is a subgroup of G .
 - $H \cap N$ is a normal subgroup of G .
 - HN/N is isomorphic to $H/(H \cap N)$.
21. (a) Let F be a field and $f(x) \in F[x]$. Show that $a \in F$ is a zero of $f(x)$ if and only if $(x - a)$ is a factor of $f(x)$.
- (b) Prove that a polynomial of degree 3 in $F[x]$ is irreducible if and only if it has no zero in F .

(2 × 5 = 10 weightage)