

Section B

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

Each question has weightage of 6

(CUCSS)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS—I

(2017 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Section A

Answer all questions.

Each question has weightage of 1.

1. Explain briefly how Laplace equation can be applied in describing electrostatic fields.
2. If A is irrotational, show that $A \times R$ is solenoidal.
3. Explain the physical significance of Unitary transformations.
4. Define outer product of tensors with an example.
5. What are Hermitian operators ? Mention any two properties.
6. What are even and odd functions ? Write corresponding Fourier series.
7. Write the Rodrigues Formula for Legendre Polynomial and deduce the value of $P_0(x)$.

Section C

8. Show that $\beta(m, n) = 2 \int_0^{\pi/2} (\sin \theta)^{2m-1} (\cos \theta)^{2n-1} d\theta$.
9. What is Dirac delta function? Mention two situations where the function can be applied.
10. Show that $J_n(-x) = J_n(x)$ for even n and $J_n(-x) = -J_n(x)$ for odd n .
11. Discuss the advantages of using the transforms of derivatives.
12. Show that the convolution of $f(x)$ and $g(x)$ is commutative.

(12 × 1 = 12 weightage)

19. Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

Turn over

Section B

Answer any two questions.

Each question has weightage of 6.

13. Describe cylindrical and spherical polar co-ordinates and show that these co-ordinate systems are orthogonal.
14. Discuss Hermitian, Unitary, and Orthogonal matrix with example. Show that eigen vectors of Hermitian matrix are orthogonal and eigen values are real.
15. (a) Explain Gram- Schmitz orthogonalisation process.
- (b) Find the distance from the point $y = (0, 0, 0, 1)$ to the subspace $V \subset R^4$ spanned by vectors $x_1 = (1, -1, 1, -1)$, $x_2 = (1, 1, 3, -1)$, and $x_3 = (-3, 7, 1, 3)$.
16. Show that $(1 - 2tx + t^2)^{-1/2}$ is a generating function of $P_n(x)$. Hence prove the following recurrence relations.
- (i) $nP_n(x) = (2n - 1)x P_{n-1}(x) - (n - 1)P_{n-2}(x)$.
- (ii) $nP_n(x) = xP_n'(x) - P_{n-1}(x)$.

(2 × 6 = 12 weightage)

Section C

Answer any four questions.

Each question has weightage of 3.

17. Consider a co-ordinate system (u, v, w) which is related to Cartesian co-ordinate system by $x = uv : y = uw : z = uv$. Obtain the metric tensor in the form of u, v, w .
18. State Stoke's theorem and verify it for the vector $A = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of a sphere $x^2 + y^2 + z^2 = 1$.
19. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

20. Define beta and gamma functions. Evaluate the integral $\int_{-1}^{+1} \left(\frac{1+x}{1-x} \right)^{\frac{1}{2}} dx$ using the properties of beta and gamma functions.
21. Show with the help of an example how Laplace transforms can be used to evaluate a definite integral.
22. Expand $f_1(x) = x^2$ for $-\pi \leq x \leq \pi$ in a Fourier series.

(4 × 3 = 12 weightage)