

D 42548

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Name.....

Reg. No.....

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2018

(CUCSS)

Mathematics

Group III-MT4 E11—GRAPH THEORY

(2016 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A (Short Answer Type)

Answer all questions.

Each question has weightage 1.

1. Prove that a connected graph is a tree if and only if every edge is a cut edge.
2. Prove that every connected graph contains a spanning tree.
3. Draw any two non-isomorphic spanning trees of $K_{2,3}$.
4. Let G be a 2-connected graph. Prove that any two vertices of G lie on a common cycle.
5. Give a graph with v vertices in which $v - 2$ vertices are cut vertices.
6. Let G be a block with $v(G) \geq 3$. Prove that any two edges of G lie on a common cycle.
7. If G is Hamiltonian, prove that $\omega(G - S) \leq |S|$ for every non-empty proper subset S .
8. If G is a simple Hamiltonian graph, prove that its closure is Hamiltonian.
9. Find all nonhamiltonian simple graphs with 5 vertices and 7 edges.
10. Verify whether a 5-regular bipartite graph has a perfect matching.
11. If G is loopless with $\Delta = 3$, then show that $\chi' \leq 4$.
12. If G is a simple planar graph with $v \geq 3$ then prove that $\varepsilon \leq 3v - 6$.
13. If G is a simple planar graph, show that $\delta \leq 5$.
14. With usual notations, prove that $\chi \leq \Delta + 1$ for any graph G .

(14 × 1 = 14 weightage)

Turn over

Part B (Paragraph Type)

Answer any **seven** questions from the following ten questions.

Each question has weightage 2.

15. Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.
16. Show that a graph G is 2-connected if any two vertices of G are connected by two internally disjoint paths.
17. Let T be a spanning tree of a connected graph G and e be any edge of T . Show that the cotree \bar{T} contains no unique bond of G .
18. Show that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.
19. If G is a simple graph with $v \geq 3$ and $\delta \geq \frac{v}{2}$, show that G is Hamiltonian.
20. Define closure of a simple graph. Is it true that the closure of a Hamiltonian graph is Hamiltonian? Prove your claim.
21. In a bipartite graph prove that the number of edges in a maximum matching is the number of vertices in a minimum covering.
22. With usual notations, show that $\pi_k(G) = \pi_k(G - e) + \pi_k(G \cdot e)$.
23. If G is a k -critical graph with 2-vertex cut $\{u, v\}$, then prove that $d(u) + d(v) \geq 3k - 5$.
24. If G is a connected plane graph prove that $v - \varepsilon + \phi = 2$.

(7 × 2 = 14 weightage)

Part C (Essay Type)

Answer any **two** questions from the following four questions.

Each question has weightage 4.

25. With usual notations, show that $T(K_n) = n^{n-2}$.
26. Let G be a simple graph with degree sequence (d_1, d_2, \dots, d_v) where $d_1 \leq d_2 \leq \dots \leq d_v$ and $v \geq 3$. If there is no m less than $\frac{v}{2}$ for which $dm \leq m$ and $d_{v-m} < v - m$, prove that G is Hamiltonian.
27. For any two integers $k \geq 2$ and $l \geq 2$, with usual notations, prove that $r(k, l) \leq r(k, l-1) + r(k-1, l)$.
28. If G is a connected simple graph and is neither an odd cycle nor a complete graph then show that $\chi \leq \Delta$.

(2 × 4 = 8 weightage)