D 141714 (Pages : 2) Name..............................

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SECOND SEMESTER (CUFYUGP) DEGREE EXAMINATION, APRIL 2026

Statistics

STA 2MN 101—PROBABILITY THEORY—I

(2024 Admission onwards)

Time : Two Hours Maximum Marks : 70

Section A

All *questions can be answered.*
Each question carries 3 *marks. (Ceiling :* 24 *marks)*

1. Define Probability mass function of a random variable X.
2. If $F_x(x)$ is the distribution function of X, mention any three of its properties.
3. For a random variable X, prove that $E(X^2) \geq [E(X)]^2$.
4. Define moments. What are its uses ?
5. Mean of an exponential random variable X is 2. Obtain P(X = 2).
6. If X is a random variable following normal distribution with mean 5 and variance 4. Find P(X > 5).
7. Define the coefficient of determination.
8. Define the regression coefficients.
9. Define a statistic. Give an example.
10. What is the relation between Chi-square and F distributions ?

Section B

All *questions can be answered.*
Each question carries 6 *marks. (Ceiling :* 36 *marks)*

11. Define moment generating function. If $M_x(t)$ is the m.g.f., of X, for two real numbers a and b show that $M_{aX+b}(t) = e^{bt} M_x(at)$.
12. Obtain the mean and variance of X following B(n, p).
13. If the mean and variance of a random variable following rectangular distribution are 8 and 12. Find the distribution.

Turn over

14. If X and Y are independent random variables following normal distribution N(3, 9) and N(4, 16) respectively, then find (i) P(X + Y < 5) ; (ii) P(X + Y > 8).

15. Define simple correlation between two variables X and Y. How is it measured ? Explain.

16. Prove that the point of intersection of the two regression lines is $\left(\bar{x}, \bar{y}\right)$.

17. Obtain the probability distribution of the mean of the random sample of size n taken from a normal population $N\left(\mu, \sigma^2\right)$.

18. Obtain the m.g.f. of X following Chi square distribution with n degrees of freedom.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

19. Define Poisson distribution. Obtain the mgf and hence the mean of X following Poisson distribution with parameter λ. For a Poisson random variable X, if P(X = 1) = P(X = 2), obtain P(X > 3).

20. Define (i) Sampling distribution ; (ii) t-distribution ; (iii) covariance between X and Y ; (iv) Obtain the covariance between X and Y using the data (X, Y) = (4, 7), (5, 9), (6, 11), (7, 14), (8, 18).

(1 × 10 = 10 marks)