C 80273 (Pages : 3) Name.....................................

Reg. No.....................................

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS—UG)

Mathematics

MAT 6B 13 (E01)—GRAPH THEORY

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define parallel edges.
2. Which type of graphs are complete graphs ?
3. Define degree of a vertex.
4. What do you mean by a spanning subgraph ?
5. When do we say that a graph is connected ?
6. Define a bridge in a graph.
7. How many different spanning trees does a complete graph K_n have ?
8. Define a cut vertex.
9. If G is a simple graph, define $k(G)$, the connectivity of G.
10. Define an Euler tour.
11. What is a maximal non-Hamiltonian graph ?
12. Define a plane graph.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Prove first theorem of graph theory.
14. If G is a k-regular graph, where k is an odd number, show that the size of the graph is rk for some integer k.
15. Show that in any tree with at least two vertices, there will be at least two vertices of degree 1.

Turn over

16. Prove that every tree with at least two vertices is a bipartite graph.

17. If the closure $c(G)$ of a graph G is Hamiltonian, show that G is Hamiltonian.

18. Prove that every subgraph of a planar graph is planar.

19. Show that K_5 is non-planar.

20. Show that a Hamiltonian graph G has no cut vertex.

21. Find the condition on n, such that K_n is Eulerian.

22. Write the adjacency matrix of the union of K_3 and P_3.

23. Give an example of a graph which is Eulerian but non-Hamiltonian.

24. Verify Eulers formula for K_4.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions from the following.*
Each question carries 5 marks.

25. Let G be a graph with n vertices t of which have degree k and the remaining have degree $k+1$. Prove that $t = (k+1)n - 2|E(G)|$.

26. A simple graph is said to be self-complementary if it is isomorphic to its own complement. Show that $|V(G)| = 4k$ or $|V(G)| = 4k+1$ for any self complementary graph G.

27. Prove that for any connected graph G, $rad(G) \leq diam(G) \leq 2\,rad(G)$.

28. Show that in any tree T, there is exactly one path joining any two distinct vertices.

29. If $|V(G)| \geq 2$, show that G has at least two vertices which are not cut vertices.

30. If G is a graph in which $d(v) \geq 2$ for all $v \in V(G)$, show that G has a cycle.

31. Show that $K_{3,3}$ is non-planar.

32. If G is a connected graph, show that it is a tree if every edge of G is a bridge.

33. Prove or disprove : If u is an odd vertex in a graph G, then there is path in G from u to another odd vertex $v \in V(G)$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (a) State and prove a characterization theorem for a vertex in a connected graph to be a cut vertex.

    (b) Show that a graph G is connected if and only if it has a spanning tree.

35. (a) Let G be a simple graph with n vertices and $\overline{G}$ be its complement. Show that $d_G(v) + d_{\overline{G}}(v) = n - 1$. Moreover, if G has exactly one even vertex, how many odd vertices does $\overline{G}$ have ?

    (b) Show that an edge e of a graph G is a bridge if and only if e does not belong to any cycle in G.

36. (a) If G is a simple graph show that either G or $\overline{G}$ is connected.

    (b) Let G be an acyclic graph with n vertices and k components, determine the number of edges and prove the result.

(2 × 10 = 20 marks)